

WELDED TAPERED GIRDERS



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Figure 1

Shepherd Machinery Company Showroom, Whittier, Calif.
The Ted R. Cooper Company, Designers and Construction Engineers

INTRODUCTION

In recent years tapered girders fabricated by welding plates together have become increasingly popular in the framing of roofs over comparatively large areas where it is desirable to either minimize the number of interior columns or to eliminate them altogether, dependent upon the width of the building. The two halves of the web are produced from wide plates, with little or no waste of material, by making one longitudinal diagonal cut. These halves are then rotated and spliced to give the maximum depth at mid-span. When camber is required it may be obtained very simply by skewing the two halves slightly between their abutting edges before making the splice.

Roof loads being relatively light, tapered girders may generally be fabricated from plates the thickness of which is limited only by availability and the maximum web depth-thickness provision of the A.I.S.C. Specification.

When the girders are used with the sloping flange up, their taper in both directions from the ridge provides the slope that may be required for drainage. Furthermore, by varying the end depth of successive girders the deck can be canted to drain toward roof boxes in the valleys between adjacent gabled spans and at flanking parapet walls, thereby eliminating the necessity for crickets.

For flat roofs the girders are inverted, the tapered

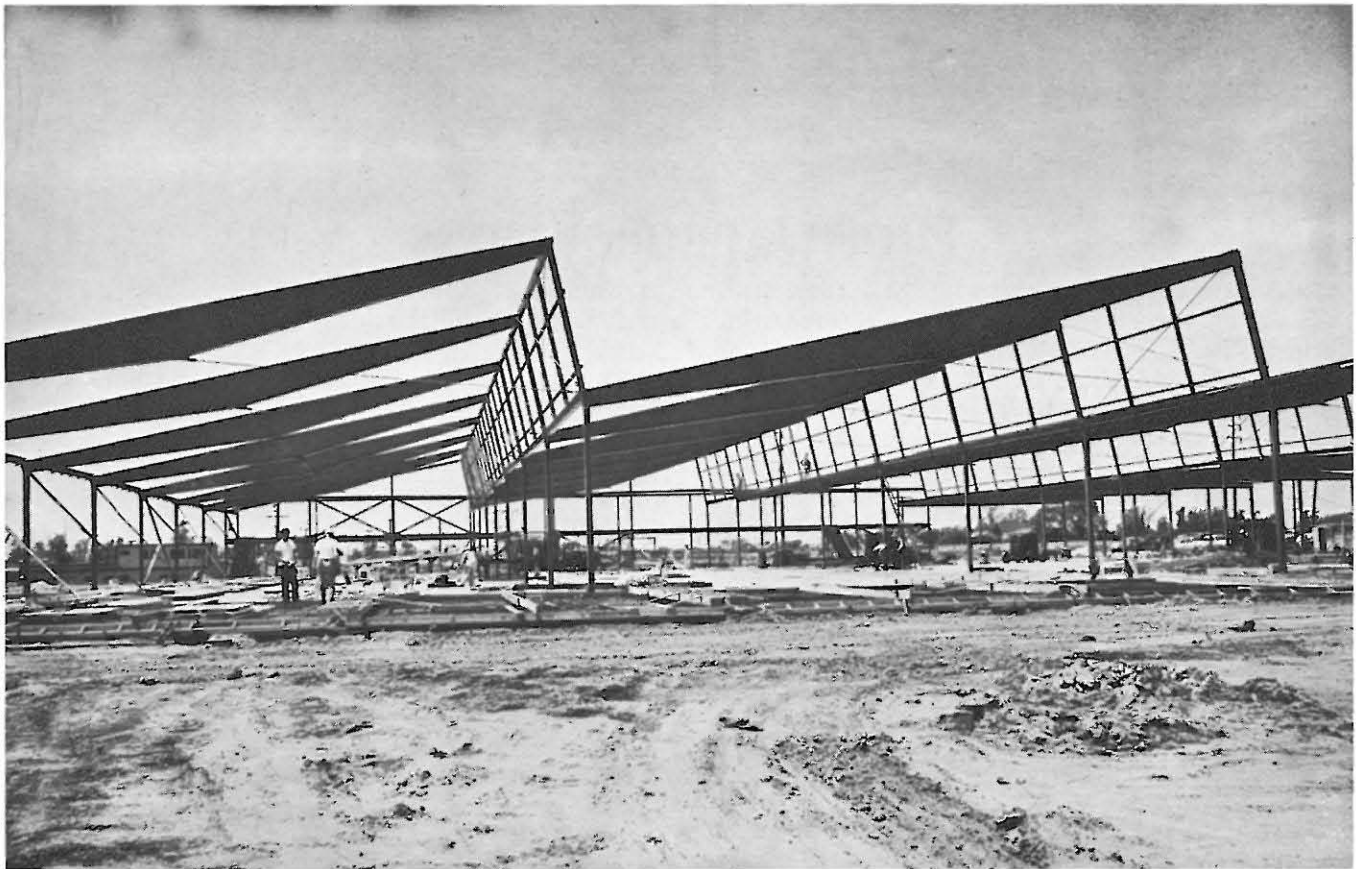


Figure 2

Plant for Minneapolis-Honeywell Regulator Company, Gardena, Calif.
Kenneth H. Neptune, Architect; Richard R. Bradshaw, Structural Engineer

flange being down. Some other roof designs frequently call for a gable ridge in the center span of three spans across the width of the building. In such a case inverted girders are used in the outside spans thereby continuing the same slope of decking to the walls.

There are also additional advantages. Economy is realized in diminished overall height of exterior walls as a result of the reduced depth of web at the ends of the girders. Also when used as the principal carrying members for ordinary joisted roof con-

struction above, and a fire retardant ceiling below, tapered girders provide the tight draft stops required by many building codes as a means of subdividing the attic space.

There is a diversity of roof framing systems possible with tapered girders. Several systems are illustrated in Figs. 1, 2, 3 and 6. That the resulting designs are competitive with other constructions is evidenced by the growing number of buildings for which engineers and architects have specified their use.

CRITICAL SECTION

Since the section modulus of a tapered girder is itself a variable, dependent upon the taper, the location of the critical or design section at which the bending stress will be maximum, is not always immediately apparent. Seldom is it located at the point of maximum moment, the exception being the case of a girder designed for concentrated mid-span loading.

In the case of concentrated loading, a critical section will occur at one or at most two load points

where there is a sharp break in the moment diagram.

In the design of uniformly loaded tapered girders the critical section is often assumed to be located at quarter-span. However, there are limits within which the taper must then be kept. For example, if the taper of a girder designed on this assumption were reduced to zero it would be over-stressed at mid-span by one-third of the allowable value, since the quarter-span moment is but three-fourths that at mid-span.

PROFILE PROPORTIONS

To obtain the utmost in bending resistance from a symmetrical I-shape profile of given area, the total area of its flanges should be equal to the area of its web. Since, however, a $\frac{1}{16}$ in. increase in web plate thickness could increase the web area of a tentatively selected profile by as much as one third, obviously no hard-and-fast flange-web area rule can be laid down. Fortunately no such rule is necessary. The ratio of flange to web area can be varied, within reasonable limits, with only minor effect upon the bending efficiency of a given profile area. Nevertheless, equality of web and flange area does afford a useful guide to economy. It is, therefore, reflected in the formulas which follow.

These formulas have been considerably simplified by substituting the approximately correct expression

$$S = A_f h + \frac{h^2 t}{6}$$

for one of the precise but much more complex formulas for S . Any resulting error, as applied to the design of tapered girders fabricated from flat plates, will be less than 2 percent and hence of little conse-

quence in the final selection from available plate sizes.

The design of a tapered girder to suit a given span and set of loads is often executed using a cut-and-try procedure. In most cases the economical proportions for the profile *at the critical section* can be obtained with a minimum of effort using the following procedure:

1. Compute the required web depth as

$$h = \sqrt[3]{\frac{18Mn}{J}} \quad (1)$$

where n is a tentatively assumed value for the web depth-thickness ratio at the section under study, such that the web would be as thin as possible and

* This expression, as well as those given by Eqs. (2) and (3), is rigorously correct only when the area of the web *at the critical section* is equal to the total flange area. However, values of h derived from its use afford good economy, even when the flange and web areas established in subsequent steps of the outlined procedure deviate somewhat from this relationship.

still satisfy all specification requirements anywhere throughout the girder. If the critical section is at mid-span n will generally be taken as 170; if at or near quarter-span it can be taken as somewhere between 95 and 115.

2. Divide h by n to obtain tentative web thickness t' , and compute tentative flange area as

$$A_{F'} = \frac{ht'}{2}$$

3. Select available web plate thickness t nearest to t' , making sure that, at the desired taper, the web area at the supports will be adequate for the given shear and that, at mid-span, h/t will be no greater than 170.

4. Compute required flange area as

$$A_F = A_{F'} - \frac{h(t - t')}{6} \text{ or } A_{F'} + \frac{h(t' - t)}{6}$$

5. Select available flange plate size whose area is equal to or greater than A_F .

When for any reason, the actual web thickness t has already been established, Eq. (1) can be modified to read

$$h = \sqrt{\frac{18M}{ft}}^* \quad (2)$$

Here the required flange area may be computed at once as

$$A_F = \frac{ht}{2}$$

In the case of a uniformly loaded tapered girder, designed on the assumption that the critical section is at quarter-span and that the desired value of n at mid-span will be 170, Eq. (1) can be expressed as

* Note that the index to the radical in this case is 2, instead of 3 as in Eq. (1).



Figure 3

Industrial Building for Container Corporation of America, Bechtel Corporation, Engineers

$$h = 5.72 \sqrt[3]{\frac{wL^2}{f}} \quad (3)$$

The corresponding tentative web thickness will be

$$t' = \frac{h}{111}$$

The slope or "bevel" of the taper which will place the critical section at quarter-span of a uniformly loaded girder proportioned in accordance with Eq. (3) is given by the formula

$$\phi' = \frac{0.45wL}{3A_F + ht} \quad (4)$$

If a steeper or flatter taper is used the critical section will not be at quarter-span. It will be nearer mid-span if the slope is too flat and nearer the end support if it is too steep. The magnitude of the bending stress at the actual critical section can easily be determined with the aid of Fig. 4. With the ratio r , equal to the required taper ϕ divided by the value of ϕ' computed using Eq. (4), a factor is obtained by which the quarter-span bending stress can be corrected to give the maximum value anywhere in the girder.

A similar procedure can usually be followed to obtain the maximum bending stress in a symmetrical girder having a given taper and required to support a combination of uniform and concentrated loads. Using Eq. (1) (or Eq. (2) if the web thickness is fixed by shear or depth-thickness limitations) design a tentative quarter-span profile and compute the bending stress using this profile. The taper which would place the critical section at quarter-span can be computed as

$$\phi' = \frac{1.8V}{3A_F + ht} \quad (5)$$

where V is the shear at the section for which the profile has been tentatively designed. As in the case of girders having only uniform loading, enter Fig. 4 with the ratio

$$r = \frac{\phi}{\phi'}$$

to obtain a correction factor for the bending stress computed for this section, and modify the flange area if necessary to avoid overstress. The correction factor thus obtained, in this case, may err

somewhat on the conservative side, but generally not to any wasteful extent.

Note that there are two values for ϕ' when the critical section is assumed to be at a concentrated load, different only in proportion to the shear either side of this load. When such a load is located adjacent to quarter-span, time may be saved in the design if the critical section is tentatively taken at the load point.

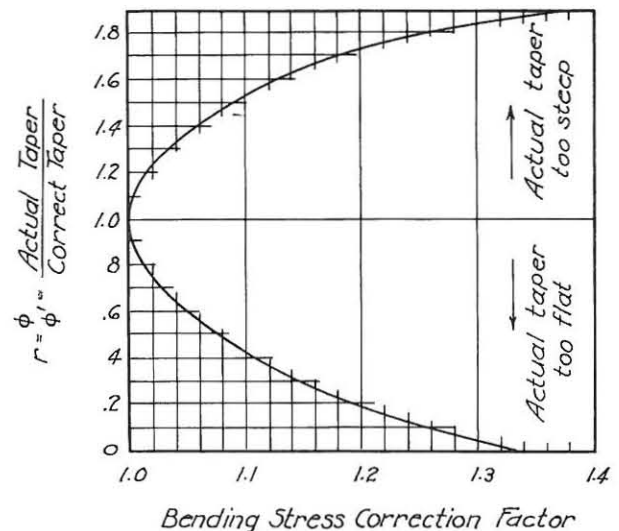


Figure 4

Occasionally it may be required to design a girder having such a steep taper that the smallest possible quarter-span depth will be substantially greater than the most economical one for the given loading indicated by use of Eqs. (1) and (2). In such cases a trial flange area can be obtained, as in Problem 5, by subtracting the bending strength of the web from the total requirement at a section estimated to be near critical. The adequacy of this flange area is then investigated by computing the moment, section modulus and bending stress at intervals either side of the previously assumed section.

STIFFENERS

The A.I.S.C. Specification (Sect. 26(e)) recommends the use of stiffeners at unframed ends, at points of concentrated loads, and when the unit web shear stress v , expressed in kips per sq. in., exceeds $\frac{64,000}{(h/t)^2}$.

Ordinarily, intermediate stiffeners are not required on tapered girders since the loads they support are relatively light for any given span. If necessary in order to avoid their use, it is generally more economical to increase the web thickness. The limiting web depth, h_s , below which stiffeners will not be required, can be determined from the formula

$$h_s = \frac{64,000t^3}{V} \quad (6)$$

Using the appropriate value for $64,000t^3$ obtained from Table I, and the total calculated shear at any given cross-section, the need for intermediate stiffeners at that section can be determined by comparing its actual web depth with the value derived from Eq. (6).

The relatively light reactions from intermediate beams and purlins are not considered as coming within the meaning of "concentrated" loads, for which stiffeners are required. Such stiffeners will be needed, however, under the following circumstances:

(1) When the end of a beam resting on the top flange of a girder is stopped so far short of

TABLE I

Web thickness t — in.	$64,000t^3$
$\frac{3}{16}$	422
$\frac{1}{4}$	1,000
$\frac{5}{16}$	1,950
$\frac{3}{8}$	3,380
$\frac{7}{16}$	5,560
$\frac{1}{2}$	8,000
$\frac{9}{16}$	11,400
$\frac{5}{8}$	15,600
$\frac{3}{4}$	27,000
1	64,000

its centerline as to produce serious transverse bending stresses in the girder flange unless it is supported by stiffeners.

(2) When the reaction of a beam resting on the top flange would otherwise produce serious crippling stresses in the girder web at its juncture with the top flange, the stress analysis being made in accordance with Sect. 26(h) of the A.I.S.C. Specification.

(3) When the beams or purlins frame into the girder web and it is desired to brace the girder against any tendency to roll during erection. (See Fig. 1.)

DEFLECTION

No simple formula has ever been written for computing the deflection of a tapered girder. It is dependent not only upon the type of loading and moment of inertia at mid-span, but also upon the steepness of the taper as well.

In general it can be stated that the deflection of a girder, designed in accordance with the procedure outlined on pp. 4 and 5 will be no greater, and often substantially less, than the deflection of a straight girder economically designed for mid-span moment, the possible exception being a tapered girder designed to support only a mid-span concentrated load.

A rough approximation of the deflection which a tapered girder will develop at mid-span under a given set of loads can be made using the usual formulas* and assuming a constant value for I equal to

the average for the given girder.

If it is necessary to compute the deflection accurately this may be done by summation of increments, as shown in Fig. 7, using the expression

$$\Delta = \sum \frac{M_x m x}{EI_x} \quad (7)$$

where m represents increments of span length, the centers of which are distance x from the end support; M_x and I_x being, respectively, the moment and moment of inertia the same distances from the support.

Little error will result if, in order to simplify the computations the moment of inertia is taken as

$$I_x = \frac{A_f h_x^2}{2} + \frac{h_x^3 t}{12} \quad (8)$$

* See A.I.S.C. Manual pp 366-369.

WELDING

The intensity of longitudinal shear transfer between flange and web plate at any point along a built-up girder can be expressed as

$$v_w = \frac{VQ}{I}$$

where Q is the statical moment of the area of one flange with respect to the neutral axis, and v_w is in kips per lin. in. of girder when V , the applied shear at the point, is given in kips.

In keeping with the simplified approximation used in computing values for section modulus, the profile property Q/I can be written

$$\frac{Q}{I} = \frac{A_F}{S} \quad (\text{Approx.})$$

Then

$$v_w = \frac{VA_F}{S} \quad (\text{Approx.}) \quad (9)$$

If this shear is resisted by pairs of fillet welds, placed opposite one another with respect to the girder web, the effective size of these welds should not be taken as more than two-thirds of the web thickness, in order to avoid overstressing the web plate in shear at the toe of the fillets. If the welds are staggered, no reduction in effective size will be necessary. When relatively little welding is needed to provide for the shear transfer, a staggered pattern will meet the requirements of the A.I.S.C. Specification, Sect. 25(b), as to clear spacing between weld segments, with less welding than would be required with pairs of welds located opposite one another.

ACCESS HOLES

It is often necessary to provide openings in the webs of tapered girders for pipes, air conditioning ducts, and passageway within the attic space formed by the roof deck and a suspended ceiling. Except in the case of relatively small holes, the girder adjacent to these openings should be investigated for stress and stability.

Large openings, providing for the passage of a man, are generally located at mid-span. Here the depth of girder is greatest; the shear produced by the service loads is at a minimum; and, in the case of uniformly loaded tapered girders, bending stresses generally are considerably less than the maximum permitted.

When such holes must be located in regions of substantial shear, the additional local bending stresses produced by this shear must be combined with the stresses due to general bending about the neutral axis of the girder.

Fig. 5 illustrates this case. It is assumed that the local bending in the T-sections above and below the hole produces points of contraflexure in line with the vertical axis of the hole. The total shear V , at this point on the girder span, is distributed, above and below the hole, in proportion to the average depth of the tee stems,* and the moments producing local bending stresses at the sections through the

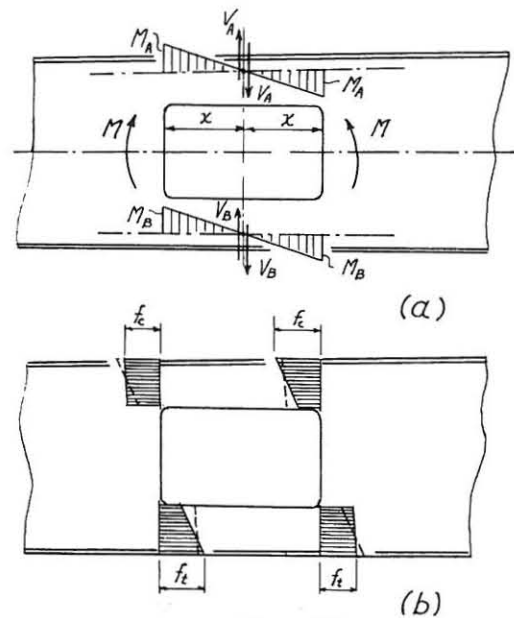


Figure 5

vertical sides of the holes are computed as

$$M_A = V_A x \text{ and } M_B = V_B x$$

The maximum stresses resulting from these moments, combined with those produced by the general

* $V = V_A + V_B$. Generally the hole will be centered approximately on the neutral axis of the girder and $V_A \cong V_B$.

bending, are shown by the shaded area of the stress diagram in Fig. 5(b).

To prevent this combined stress from exceeding the maximum permitted, it is sometimes necessary to add flange material along the upper and lower edges of the hole, thus converting the T-sections into modified I-sections. When this is the case the flange material furnished should be long enough so that it can be connected to the uncut portions of the web with sufficient welding to develop the required flange strength, outside the limits of the hole.

Even if flange material is not required in order to prevent excessive stress concentrations, it may be needed to provide lateral support along the upper edge of the hole, where the web is in compression. In such cases the provisions of Sect. 18(b) of the A.I.S.C. Specification, relating to the stems of compression tees, can be used to test the adequacy of the unstiffened edge.

A section at the vertical axis of the hole, where the free edge of the hole receives the least amount of lateral support, should be investigated. Here the only bending to be considered is that which takes place about the neutral axis of the girder. In lieu

of the properties of the actual net girder profile at this point, the moment of inertia of a hypothetical profile, comprising tees whose stem width is 16 times the girder web thickness, is used to compute bending stresses. If the maximum thus obtained comes within the permissible limit it may be concluded that lateral support for the free edge is unnecessary.

Some judgment is required as to the need for applying this rule in the case of relatively small holes where the compressive stress at the upper edge is low by reason of its proximity to the neutral axis. If the longitudinal dimension of the hole is such as to afford little support midway between the flanking uncut portions of the web, the intensity of the combined stress at the ends of the hole is likely to dictate the use of flange plates anyway. It is doubtful, therefore, whether the rule ever need apply when the depth of hole is less than say 0.4 the depth of the girder at the hole.

Because of resulting high local stress concentrations, sharp corners should be avoided when the upper and lower edges of the hole are not reinforced with flange material. Such corners should be formed to a minimum radius of 2 in.



Figure 6

Manufacturing Building for Price-Pfister Bros. Mfg. Co., Pacoima, Calif.

BRACING

The problem of lateral support for the top (compression) flange of tapered girders is no different from that of other beams and girders of similar proportions. Generally the roof deck is sufficiently rigid to function as a diaphragm, and it is only necessary to provide frequent and positive connections of the top flange to this deck. There is no economy in designing for a reduced bending stress, in accordance with the ld/bt formula, in order to permit a greater distance between bracing points at the top flange.

Because of the shallow end depth of tapered girders, their connection to supporting columns like the connection of Fink and bow-string trusses to their supporting columns, affords little resistance to horizontal forces. To insure adequate resistance to

such forces, knee braces may be required, unless the roof deck, or a positive system of bracing in the plane of the roof, is stiff enough to transmit them to adequately braced walls.

If the roof deck is employed as an essential part of the permanent bracing system, adequate provision must be made to brace the girders during construction and until the roof deck is in place and securely fastened to the supporting frame.

It is recommended that, where tapered girders are to be supported on masonry walls, they be set on wall plates, leveled to grade and grouted, and that they be secured by means of a pair of properly embedded anchor bolts as a safety measure during erection.

DESIGN PROBLEM #1

Required to design a symmetrical 60' tapered beam to support a uniform load (including its own weight) of 650 lbs. per lin. ft., assuming the top flange to be fully stayed laterally.

From Eq. (3), the quarter-span web depth

$$h = 5.72 \sqrt[3]{\frac{0.65 \times 60^2}{20}} = 28.0''.$$

$$t' = \frac{28.0}{111} = 0.252'' \quad \text{Use } \frac{1}{4} \text{ plate}$$

$$A_{r'} = \frac{28.0 \times 0.25}{2} = 3.50 \text{ sq. in.}$$

Try 2 $\text{pl}_8 - 8 \times \frac{1}{16}$ (Area one $\text{pl}_8 = 3.50$ sq. in.)

$$S = 3.50 \times 28.0 + \frac{28^2 \times 0.25}{6} = 130.7$$

$$M = \frac{3}{4} \times \frac{0.65 \times 60^2}{8} = 219 \text{ k'}$$

$$f = \frac{12 \times 219}{130.7} = 20.1 \text{ ksi (o.k.)}$$

At mid-span, let

$$h = 170 \times 0.25 = 42.5''$$

$$\phi = \frac{42.5'' - 28.0''}{15'} = 0.97 \text{ in. per ft.}$$

At the end supports

$$h = 28.0 - (15 \times 0.97) = 13.5''$$

$$V = 0.65 \times 30 = 19.5 \text{ k}$$

$$v = \frac{19.5}{13.5 \times 0.25} = 5.8 \text{ ksi (O.K.)}$$

From Eq. (4)

$$\phi' = \frac{0.45 \times 0.65 \times 60}{(3 \times 3.5) + (28.0 \times 0.25)} = 1.00 \text{ in. per ft.}$$

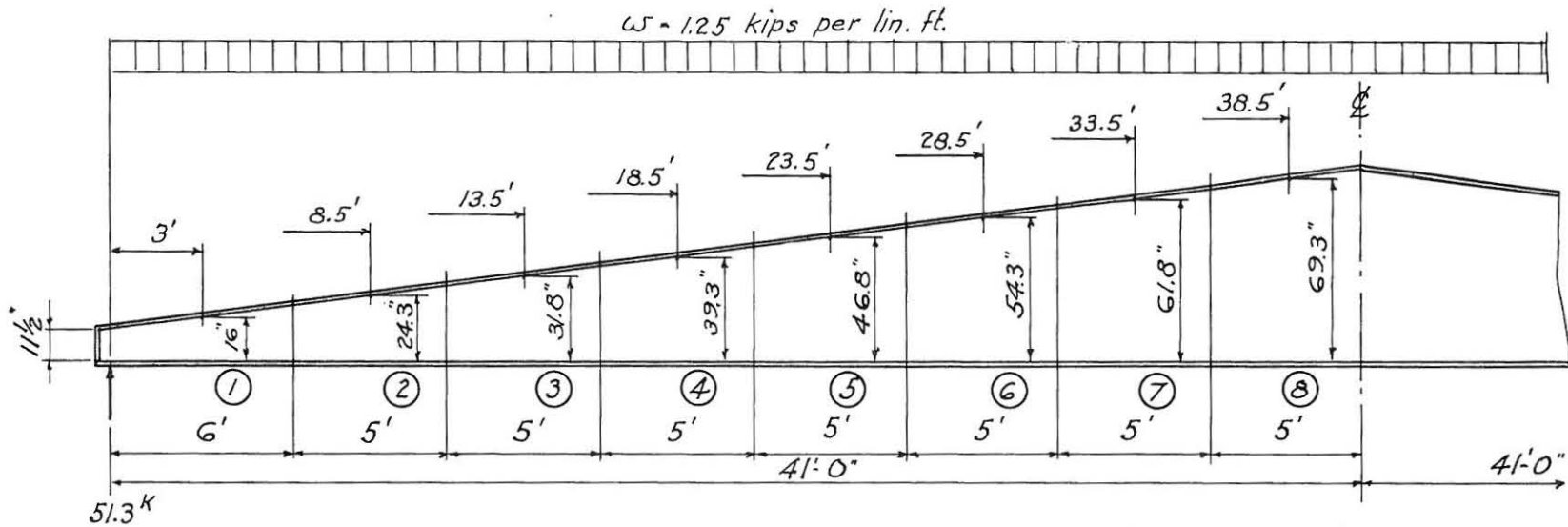
$$r = \frac{\phi}{\phi'} = \frac{0.97}{1.00} = 0.97$$

Refer to Fig. 4 and note that the required correction in computed maximum bending stress, due to the slight deviation from the taper which would place the critical section at quarter-span, is less than 1 percent.

From Eq. (6) and Table I

$$h_s = \frac{1,000}{19.5} = 51.3'' > 42.5''$$

Intermediate stiffeners are not required.



Deflection Calculations

Sect-ion	m ft.	x ft.	h in.	I in. ⁴	M kip ft.	$\frac{Mmx}{I}$
1	6.0	3.0	16.0	1,270	148	2.10
2	5.0	8.5	24.3	3,105	391	5.35
3	5.0	13.5	31.8	5,600	579	6.98
4	5.0	18.5	39.3	8,970	735	7.58
5	5.0	23.5	46.8	13,320	860	7.59
6	5.0	28.5	54.3	18,740	954	7.25
7	5.0	33.5	61.8	25,310	1,017	6.73
8	5.0	38.5	69.3	33,150	1,049	6.09

$$\sum \frac{Mmx}{I} = 49.67$$

$$M = 51.3x - 0.625x^2 \text{ kip ft.}$$

$$h = 11.5 + 1.5x \text{ in.}$$

$$\Delta_{max.} = \frac{1728}{30,000} \sum \frac{Mmx}{I} = 2.86 \text{ in.}$$

Figure 7

DESIGN PROBLEM #2

Required to design an 82' girder, tapered $1\frac{1}{2}$ in. per ft. each way from mid-span, to support a uniform load (including the weight of the girder itself) of 1.25 kips per lin. ft., assuming the top flange to have complete lateral support.

Using Eq. (3), compute the quarter-span depth as

$$h = 5.72 \sqrt[3]{\frac{1.25 \times 82^2}{20.0}} = 42.8''$$

$$t' = \frac{42.8}{111} = 0.386''$$

$$ht' = 0.386 \times 42.8 = 16.5 \text{ sq. in.}$$

$$A_{F'} = 8.25 \text{ sq. in.}$$

Req'd. difference between $h_{(\max)}$ and $h_{(\min)}$ = $1\frac{1}{2}'' \times 41' = 61\frac{1}{2}''$

At supports, $V = 1.25 \times 41 = 51.3^k$

Try $\frac{1}{16}''$ web plate

$$h_{(\min)} = \frac{51.3}{13.0 \times 0.438} = 9.0''$$

$$\frac{h_{(\max)}}{t} = \frac{9.0 + 61.5}{0.438} = 161 < 170$$

∴ Least quarter-span depth = $9.0'' + \frac{61.5''}{2} = 39.8'' < 42.9''$ ($\frac{1}{16}$ plate o.k.)

When $h = 42.8''$ and $t = 0.438''$

$$A_F = 8.25 - \frac{42.9(0.438 - 0.386)}{6} = 7.93 \text{ sq. in.}$$

Try 2 #s-16 x $\frac{1}{2}$ (Area one # = 8.00 sq. in.)

At quarter-span, let

$$h = 12\frac{1}{2}'' - (2 \times \frac{1}{2}'') + \frac{61\frac{1}{2}''}{2} = 42.3''$$

$$S = 8.00 \times 42.3 + \frac{0.438 \times 42.3^2}{6} = 469 \text{ in.}^3$$

$$M = \frac{3}{4} \times \frac{1.25 \times 82^2}{8} = 788^k'$$

$$f = \frac{12 \times 788}{468} = 20.2 \text{ ksi}$$

Using Eq. (4)

$$\phi' = \frac{0.45 \times 1.25 \times 82}{(3 \times 8.0) + (42.3 \times 0.438)} = 1.08''/'$$

$$r = \frac{\phi}{\phi'} = \frac{1.50}{1.08} = 1.39$$

From Fig. 4, correction factor = 1.06

$$f_{(\max)} = 20.2 \times 1.06 = 21.4 \text{ ksi (Too much)}$$

Req'd. $S = 469 \times 1.06 = 497$

Let A_{FF} = flange area giving S of 497 in.³ without increasing depth h .

$$42.3 A_{FF} + \frac{0.438 \times 42.3^2}{6} = 497$$

$$A_{FF} = 8.66 \text{ sq. in.}$$

Use #s-14" x $\frac{5}{8}''$ (Area one # = 8.75 sq. in.)

Calculations to obtain the maximum deflection of the girder as designed are given in Fig. 7.

DESIGN PROBLEM #3

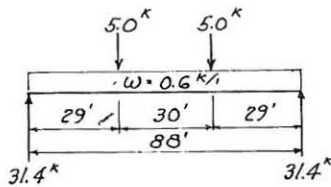


Figure 8

Required to design an 88' girder to support the loading shown in Fig. 8, making the mid-span depth 4'-0" greater than the depth at the supports. Assume complete lateral support for top flange.

Try $\frac{1}{16}''$ web plate

$$h_{(\max)} = 170 \times \frac{1}{16} = 74.4''$$

$$h_{(\min)} = \frac{31.4}{13.0 \times 0.438} = 5.5''$$

Difference = $68.9'' > 4'-0''$ (o.k.)

At quarter span

$$M = (31.4 \times 22) - \frac{0.6 \times 22^2}{2}$$

$$= 546^k'$$

$$h = \sqrt[2]{\frac{18 \times 546}{20 \times 0.438}} = 33.5''$$

$$h_{(min)} = 33.5'' - 24'' = 9.5''$$

$$h_{(max)} = 33.5'' + 24'' = 57.5''$$

$$ht = 33.5 \times 0.438 = 14.7 \text{ sq. in.}$$

$$A_F = \frac{14.7}{2} = 7.35 \text{ sq. in.}$$

Try 2 $\#$ s 12" x $\frac{5}{8}$ " (Area one $\#$ = 7.5 sq. in.)

$$S = (7.5 \times 33.5) + \frac{0.438 \times 33.5^2}{6} = 333 \text{ in.}^3$$

$$f = \frac{546 \times 12}{333} = 19.7 \text{ ksi}$$

$$V = 31.4 - (22 \times 0.6) = 18.2^k$$

Using Eq. (5)

$$\phi' = \frac{1.8 \times 18.2}{(3 \times 7.5) + 14.7} = 0.88''/'$$

$$\phi = \frac{48''}{44'} = 1.09''/'$$

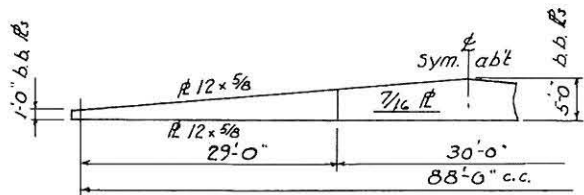


Figure 9

$$r = \frac{1.09}{0.88} = 1.24$$

From Fig. 4

Correction factor = 1.03

$$f_{(max)} = 19.7 \times 1.03 = 20.3 \text{ ksi}$$

The final design is shown in Fig. 9.



Consolidated Freightways Transfer Depot—Los Angeles, Calif.
Stiles & Robert Clement, Architects-Engineers
Note cantilever support for roof over loading platform

DESIGN PROBLEM #4

Required to design a symmetrical 40' tapered girder to support a concentrated mid-span load of 82k. Assume adequate lateral support for the top flange.

Solution A

Since the critical design cross-section is at mid-span where the moment is 820 kip ft., assume h/t as 170. Using Eq. (1)

$$h = \sqrt[3]{\frac{18 \times 820 \times 170}{20}} = 50.0''$$

$$t' = \frac{50.0}{170} = 0.294''$$

$$ht' = 50.0 \times 0.294 = 14.7 \text{ sq. in.}$$

$$A_{F'} = \frac{14.7}{2} = 7.35 \text{ sq. in.}$$

Use $\frac{5}{16}''$ web plate

$$A_F = 7.35 - \frac{50.0 (0.313 - 0.294)}{6} = 7.19 \text{ sq. in.}$$

Use 2 $\text{PL-}12'' \times \frac{5}{8}''$ (Area one $\text{PL} = 7.50 \text{ sq. in.}$)

In the given problem the moment diminishes linearly from maximum at mid-span to zero at the ends of the girder. Due to the taper, the section modulus likewise diminishes from its mid-span value to a smaller value at the ends of the girder. Unless the depth is reduced nearly to zero at the supports there will be no other section where the bending stress is as great as at mid-span. In the case of a girder supporting a single concentrated load at mid-span, the slope of the taper may be varied at will, provided that enough web area is available to take the end shear, and that h/t at mid-span does not exceed 170.

In the given problem the end shear is 41k. If h/t is not more than 70 the allowable unit shear stress is 13 ksi, in which case the depth of web plate at the supports could be as small as

$$h = \frac{41.0}{13.0 \times 0.313} = 10.1''$$

Then

$$h/t = \frac{10.1}{0.313} = 32'' < 70''$$

Fig. 10 shows a design based upon the foregoing computations.

Since the shear on the web does not drop off as h/t increases, the web should be investigated as to the need for stiffeners. Vertical stiffeners will

be placed at the concentrated load. Intermediate stiffeners will be required only where the web depth is greater than

$$h_s = \frac{1,950}{41.0} = 47.7'' \text{ (From Eq. 6 and Table I)}$$

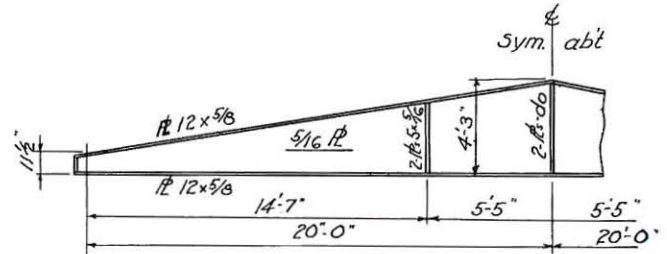


Figure 10

This is the depth of the girder at a section

$$\frac{49.8'' - 47.7''}{49.8'' - 10.3''} \times 20' \times 12'' = 12.8''$$

from mid-span.

At this section the unit web shear is

$$v = \frac{41.0^k}{47.7'' \times 0.313''} = 2.75 \text{ ksi} = 2,750 \text{ psi}$$

and the clear spacing between adjacent pairs of vertical stiffeners should not exceed

$$d = \frac{11,000 \times 0.313}{\sqrt{2,750}} = 65.7'' > 12.8''$$

Solution B

In an effort to eliminate the need for intermediate stiffeners an alternate design will be worked out using the next greater thickness of web plate ($\frac{3}{8}$ in.). Since the web thickness t is already established, Eq. (2) can be used.

$$h = \sqrt{\frac{2 \times 18 \times 820}{20 \times 0.375}} = 44.4''$$

$$ht = 44.4 \times 0.375 = 16.6 \text{ sq. in.}$$

$$A_F = \frac{16.6}{2} = 8.3 \text{ sq. in.}$$

Use 2 $\#$ s-14 x $\frac{5}{8}$ (Area one $\#$ = 8.75 sq. in.)

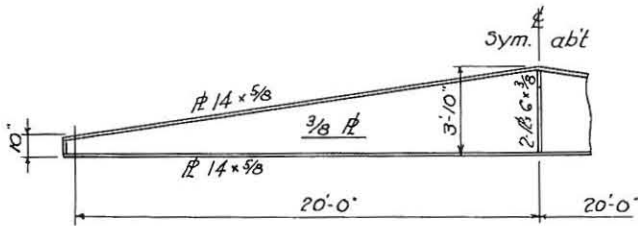


Figure 11

Minimum depth at end supports

$$h = \frac{41.0}{13.0 \times 0.375} = 8.4''$$

Fig. 11 shows this solution of the problem.

From Eq. 6 and Table I it is found that the depth of web below which intermediate stiffeners are not required

$$h_s = \frac{3,380}{41.0} = 82.4'' > 44.4''$$

Hence, intermediate stiffeners are not required.

Weight Estimate

Solution A:

2 Plates-12 x $\frac{5}{8}$ @ 51.0# x 40'	= 2040#
1 Plate -30 x $\frac{5}{16}$ @ 31.9# x 40'	= 1276
2 Plates- 5 x $\frac{5}{16}$ @ 10.6# x 4.2'	= 45
4 Plates- 5 x $\frac{5}{16}$ @ 21.2# x 3.3'	= 70
	<u>3431#</u>

Solution B:

2 Plates-14 x $\frac{5}{8}$ @ 59.6# x 40'	= 2384#
1 Plate -26 $\frac{3}{4}$ x $\frac{3}{8}$ @ 34.1# x 40'	= 1368
2 Plates- 6 x $\frac{3}{8}$ @ 15.3# x 3.7'	= 57
	<u>3809#</u>

Based upon a working stress of 20 ksi, req'd. $S = 492$. The actual section moduli for Solutions A and B are 503 and 519, respectively. Adjusting the estimated weights on the basis of required bending strength would indicate a net weight saving of about 250 lbs. in favor of the thinner web requiring intermediate stiffeners. It is problematical whether this saving in material could pay for the extra shop work, however.

DESIGN PROBLEM #5

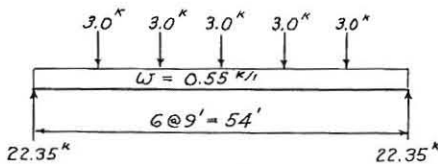


Figure 12

Required to design a 54' girder, tapered 2 in. per ft. each way from mid-span, to support the loading shown in Fig. 12. Allowable $f = 20 \text{ ksi}$.

$$h_{(max)} - h_{(min)} = 2'' \times 27 = 54''$$

Try $\frac{5}{16}$ " web plate

$$h_{(min)} = \frac{22.35}{13 \times 0.313} = 5.6''$$

$$\frac{h_{(max)}}{t} = \frac{5.5 + 54}{0.313} = 190 > 170 \text{ (Web too thin)}$$

Try $\frac{3}{8}$ " web plate

$$h_{(min)} = \frac{22.35}{13 \times 0.375} = 4.6''$$

$$h_{(max)} = 170 \times 0.375 = 63.7''$$

Let the out-to-out of flange plates be as shown in Fig. 13.

The required taper being relatively steep, the critical section will be located between the support

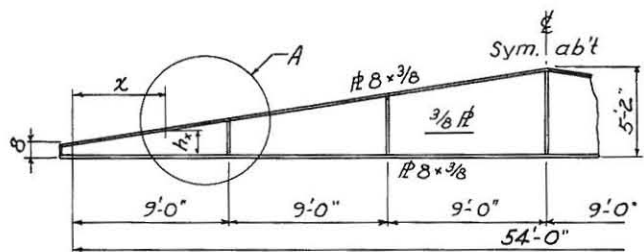


Figure 13

and quarter-span, and is likely to be at or adjacent to the concentrated load 9' from the end.

Using Eq. (1), it will be found that the economical web depth at this point is less than 21". However, the dimensions already established fix the depth 9' from the end supports as substantially more than 21".

$$\left(8'' + \frac{54''}{3} = 26''\right)$$

Hence, use of the procedure outlined on page 3 would result in a wasteful design.

Tentatively it will be assumed that the critical section leading to the most economical design for the given condition is at the concentrated load, 9' from the supports. The girder design thus developed will then be tested at frequent intervals each side of this point, identified by the circle A in Fig. 13.

At this load point $h = 25.4''$ and

$$M = (22.35 \times 9) - \frac{0.55 \times 9^2}{2}$$

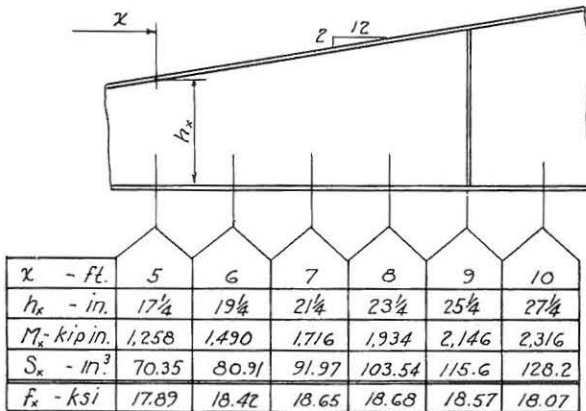
$$= 179k'$$

$$\text{Req'd. } S = \frac{179 \times 12}{20} = 107.4 \text{ in.}^3$$

$$= 25.4A_F + \frac{0.375 \times 25.4^2}{6}$$

$$A_F = 2.68 \text{ sq. in.}$$

Try 8 x 3/8 flange plates (Area one $\phi = 3.00 \text{ sq. in.}$)



Bending Stress Calculations at "A"

Figure 14

From the calculations tabulated in Fig. 14 it will be seen that the critical section for the girder, as designed, actually is slightly nearer to the end support than 9 ft. But the stress at this section

is only 0.6 percent greater than that at the load point.

Use of stiffeners at each concentrated load point in the given example was based upon the assumption that the supported purlins would be flush top with the girder, and that it would be most convenient to bolt their webs directly to these stiffeners, thus improving the stability of the girder during erection.

A satisfactory pattern of flange-to-web welding is shown in Fig. 15.

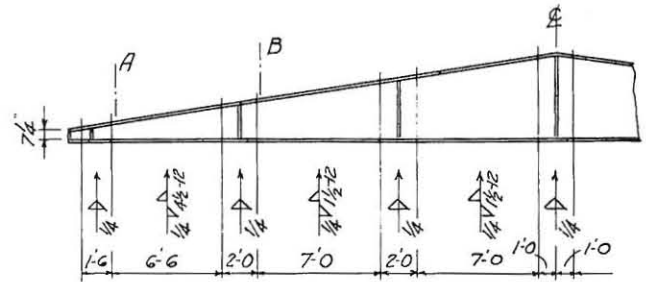


Figure 15

At section A

$$V = 22.35 - 1.10 = 21.25k$$

$$h = 7.25 + (1.5 \times 2) = 10.3''$$

and

$$S = (3.0 \times 10.3) + (0.0625 \times 10.3^2) = 37.3 \text{ in.}^3$$

Using Eq. (9)

$$v_w = \frac{21.25 \times 3.0}{37.3} = 1.71k/''$$

Req'd. length of 1/4" welds per ft., each side of web

$$\frac{12 \times 1.71}{2 \times 2.4} = 4.25''$$

Use 4 1/2" @ 12" o.c., staggered

At section B

$$V = 22.35 - 3.0 - 5.50 = 13.85k$$

$$h = 7.25 + (2 \times 10) = 27.3''$$

$$S = (3.0 \times 27.3) + (0.0625 \times 27.3^2) = 128 \text{ in.}^3$$

$$v_w = \frac{13.85 \times 3.0}{128} = 0.32k/''$$

Req'd. length of 1/4" welds per ft., each side of web

$$\frac{12 \times 0.32}{2 \times 2.4} = 0.8''$$

Use 1 1/2" @ 12" o.c., staggered

DESIGN PROBLEM #6

Required to provide 24 in. high by 36 in. wide access holes in the web of the girder studied in Problem 2, centering them 12 ft. each side of mid-span.

To simplify the calculations, and to impair the overall strength of the girder as little as possible, the holes will be centered vertically approximately on the neutral axis of the girder web, forming T-sections approximately 15½" deep. The moment of inertia of these tees, taken about their x -axis located 3.62" from the back of the flange, is 344 in.⁴. Deducting the web area removed by the hole, the moment of inertia of the girder at the vertical edge of the hole towards the support (nearest the critical section where the general bending stresses are largest) is 16,360 in.⁴. At this point the depth of the girder is 52¼" and the moment of the applied loading is 11,240k".

The shear produced by the loading at the vertical centerline of the holes is 15 kips. Divided equally above and below the hole, this produces a local bending moment in each T-section which is maximum at transverse sections coinciding with the vertical sides of the holes and equal to 135k".

Based upon these data the maximum combined bending stress, at the outer face of the flanges,

can be computed as

$$\frac{11,240 \times 26.13}{16,360} + \frac{135 \times 3.62}{344} = 19.37 \text{ ksi}$$

At the upper and lower edges of the hole the maximum combined stress is

$$\frac{11,240 \times 12}{16,360} + \frac{135 \times (15.50 - 3.62)}{344} = 12.90 \text{ ksi}$$

Flange material is not required at the upper and lower edges of the hole since the combined stress is not excessive.

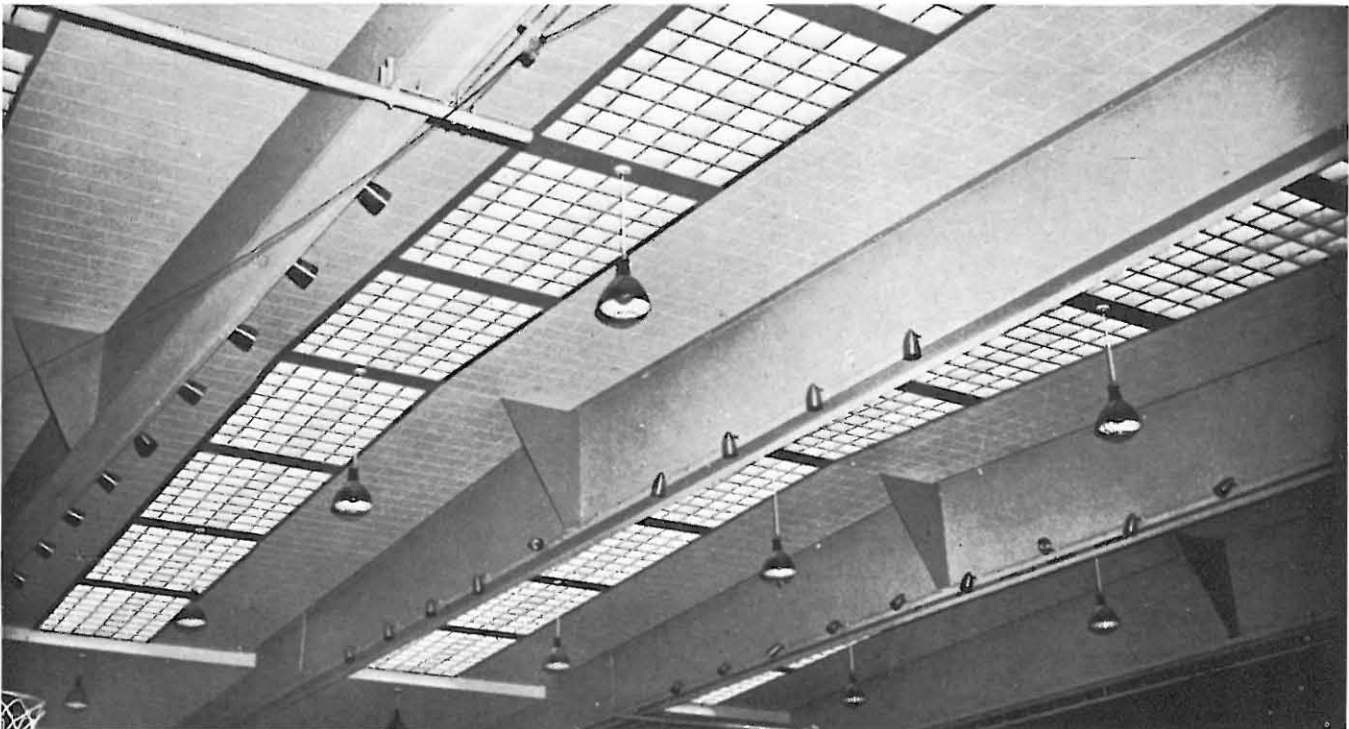
Assume, now, that the moment of the applied loads, at the centerline of the holes, must be resisted entirely by T-sections whose stems are not more than 16 x 7/16", or 7" deep.

The moment of inertia of the resulting hypothetical net girder section (55" deep, with a hole 39¾" deep) would be 16,310 in.⁴.

Based on this assumption, and a calculated moment at the hole centerline of 11,500k", the maximum bending stress would be

$$\frac{11,500 \times 27.50}{16,310} = 19.4 \text{ ksi}$$

No stiffeners to provide lateral support to the compression edge of the hole are required.



Roof over high school gymnasium, Rosemead, California, Kistner, Wright & Wright, Architect-Engineers

DESIGN PROBLEM #7

Required to provide a 20 in. high by 24 in. wide hole, centered 2 ft. from mid-span in the web of the girder shown in Fig. 10.

Calculations of the type developed in Problem 6 will show that the combined bending stress at transverse sections at the sides of the hole are excessive unless flange material is added.

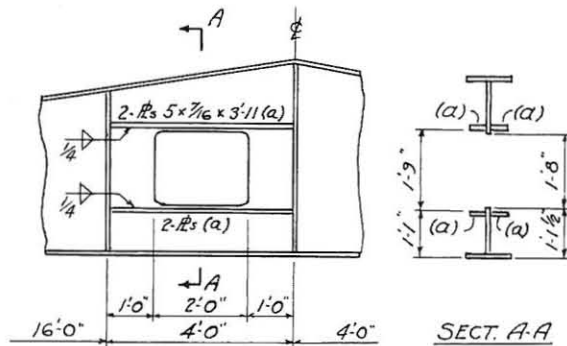


Figure 16

Try adding pairs of $5 \times \frac{7}{16}$ flange plates as indicated in Fig. 16.

The properties of the I-shape profiles above and below the hole are computed in Table A.

The moment of inertia of the net girder section can be computed as

$$2 \left[(15.90 \times 18.05^2) + 500 \right] = 11,360 \text{ in.}^4$$

At the centerline of the hole

$$V_A = V_B = 20.5^k$$

and

$$M_A = 20.5^k \times 12'' = 246^k''$$

$$M = 41^k \times 18' \times 12 = 8,860^k''$$

In the girder flanges

$$f_{(\max)} = \frac{8,860 \times 24}{11,360} + \frac{246 \times 5.45}{500} = 21.4 \text{ ksi}$$

At the edge of the hole

$$f_{(\max)} = \frac{8,860 \times 10.5}{11,360} + \frac{246 \times 8.05}{500} = 12.2 \text{ ksi}$$

Details covering the fabrication of the flange material required to reinforce the hole against excessive bending stresses are shown in Fig. 16.

TABLE A

Element	A	x	Ax	d	Ad ²	I _o
1 Plate - 12 x $\frac{5}{8}$	7.50	.31	2.3	5.14	198.1	—
1 Plate - 12 $\frac{7}{8}$ x $\frac{5}{16}$	4.02	7.06	28.4	1.61	10.4	55.8
2 Plates - 5 x $\frac{7}{16}$	4.38	12.78	56.0	7.33	235.3	—
	<u>15.90</u>		<u>86.7</u>		<u>443.8</u>	<u>55.8</u>
$d = \frac{86.7}{15.90} = 5.45''$				$I = \sum Ad^2 + \sum I_o = 500 \text{ in.}^4$		

NOMENCLATURE

- A_F — Actual or required area of one flange—sq. in.
 A_F — Tentatively assumed area of one flange—sq. in.
 E — Modulus of elasticity
 I — Moment of inertia—in.⁴
 L — Span length—ft.
 M — Moment due to loading—kip ft. unless otherwise noted
 Q — Moment of the area of one flange with respect to neutral axis—in.³
 S — Section Modulus—in.³
 V — Shear due to loading—kips
 f — Bending stress—kips per sq. in.
 h — Depth of web plate at section under consideration—in.
 h_s — Limiting web depth below which stiffeners are not required
 m — Increment of span length
 n — Web depth-thickness ratio— h/t
 r — Ratio ϕ/ϕ'
 t — Actual web thickness—in.
 t — Tentatively assumed web thickness
 v — Unit shear stress in web—kips per sq. in.
 v_w — Unit shear stress to be taken by flange-to-web welds—kips per lin. in.
 w — Uniform load—kips per lin. ft.
 x — Distance from support to any particular section
 ϕ — Actual girder taper—in. per lin. ft.
 ϕ — Tentatively assumed girder taper—in. per lin. ft.