DESIGN OF AXIALLY LOADED COMPRESSED ANGLES

Theodore V. Galambos

Department of Civil and Mineral Engineering
500 Pillsbury Drive, SE
University of Minnesota
Minneapolis, MN 55455

INTRODUCTION

Section E3 in the AISC LRFD Specification (Ref. 1) states that

"Singly symmetric and unsymmetric columns, such as angle or tee-shaped columns, and. .... may require consideration of the limit states of flexural-torsional and torsional buckling."

This requirement demands a fairly complicated procedure for the design of a common structural element which had been designed previously by the much simpler method of flexural buckling.

This paper demonstrates on the basis of analysis and experiment that many angle-columns can be designed as before by the method of minor axis buckling, and that the present AISC procedure yields conservative results for shorter columns. It is also shown that the Q-factor method, which accounts for local buckling of the outstanding legs of the angles, can be replaced by an effective width approach which unifies the process of how stiffened and unstiffened elements are designed (see Ref. 2). Finally an LRFD procedure is presented for the design of axially loaded angle columns.

ELASTIC BUCKLING

Flexural-torsional buckling involves both lateral translation and twisting of the cross section. For an unsymmetric shape, such as an unequal-leg single-angle column, the two modes of flexural buckling (i.e., about the z and u axes, see Fig. 1)

$$P_{ez} = \frac{\pi^2 EI_z}{L^2} \tag{1}$$

$$P_{eu} = \frac{\pi^2 EI_u}{L^2} \tag{2}$$

and the mode of torsional buckling

$$P_{et} = \left(\frac{\pi^2 EC_w}{L^2} + GJ\right)\left(\frac{1}{r_o^2}\right) \tag{3}$$

are totally coupled through the cubic equation

$$(P_e - P_{ez})(P_e - P_{eu})(P_e - P_{et}) - P_e^2\left[(P_e - P_{cz})\left(\frac{u_o}{r_o}\right)^2 + (P_e - P_{eu})\left(\frac{z_o}{r_o}\right)^2\right] = 0 \qquad (4)$$

For a singly symmetric section, such as an equal-leg single-angle or a double -angle column, Eq. 1 decouples because the shear center coordinate $z_o = 0$. In the case of double symmetry, such as for starred-angle columns, all three equations decouple since both $z_o = 0$ and $u_o = 0$. Equations 1, 2 and 3 are taken from Appendix E3 of the AISC LRFD Specification (Ref. 1), and the various parameters are identified in the nomenclature at the end of this paper.

The non-dimensional relationships between P_e, P_{et}, P_{ez} and p_{eu} and the slenderness parameter

$$\lambda_c = \frac{L}{r_z\pi}\left(\frac{Fy}{E}\right)^{\frac{1}{2}} \qquad (5)$$

are illustrated in Fig. 2 for an equal-leg single-angle simply supported column with a plate slenderness ratio $D/t = 12$. Also shown on the plot is the elastic plate-buckling load

$$P_{lb} = \frac{\pi^2 Ek}{12(1-\nu^2)}\left(\frac{t}{D}\right)^2 \qquad (6)$$

For short columns the elastic plate buckling load controls, while for long columns flexural buckling about the z-axis controls. Flexural-torsional buckling governs in the intermediate slenderness range.

INELASTIC BUCKLING

An examination of Fig. 2 shows that for the particular example of $D/t = 12$, the coupling action takes place well above the yield point of the steel. For a realistic assessment therefore it is necessary to consider inelastic behavior. A thorough study of this problem was made by Kitipornchai and Lee (Ref. 3) in 1986, using the finite element method. The AISC LFRD Specification (Ref. 1, Appendix E3) uses the tangent-modulus approach where the inelastic buckling load P_i is determined from Eq. 4 by replacing P_e by P_i, and P_{ez}, P_{eu}, P_{et} by P_{iz}, P_{iu}, P_{it}, respectively, where

$$P_{iz} = P_{et}\tau \qquad (7)$$

$$P_{iu} = P_{eu}\tau \qquad (8)$$

$$P_{it} = P_{et}\tau \qquad (9)$$

The tangent modulus ratio $\tau = E_t/E$, and it is based on the tangent modulus implied in the LRFD column curve in Sec. E2 of Ref. 1 (Ref. 4):

$$\tau = -2.389\ p\ \ln(p) \text{ for } 0.39 \le p \le 1.00 \tag{10a}$$

$$\tau = 0.877 \text{ for } 0 \le p < 0.39 \tag{10b}$$

where $p = P_i/P_y$ (11)

Inelastic column curves based on this approach are shown in Figs. 3, 4 and 5 for equal-leg (Fig. 3) and unequal-leg (Fig. 4) single-angles and double-angles (Fig. 5). Curves are shown for flexural buckling about the minor axis (FB) and for flexural-torsional buckling (FTB) where the yield stress is not affected by plate local buckling ($Q = 1$) and where the yield stress is reduced to QF_y, as required in the AISC LRFD Specification. Flexural buckling is always calculated with QFy. From these figures it can be seen that there is a substantial reduction from flexural strength due to flexural-torsional buckling.

The tangent modulus approach as discussed above assumes that the elastic modulus E and the shear modulus G vary as τ in the inelastic range. This assumption is a conservative one (Refs. 5 through 8), and it has been argued that the shear modulus G does not change at all when the member is yielded in compression an instant just prior to buckling. Equation 9 then can be expressed as

$$P_{it} = \left[\frac{\pi^2 E \tau C_w}{L^2} + GJ\right]\left[\frac{1}{r_o^2}\right] \tag{12}$$

The effect of this modification is evident in the column curves of Figs. 6 through 10. These curves are typical representatives of all angle columns. From Figs. 6 and 7, and from other similar plots of other single angles, it is evident that flexural-torsional buckling (FTB) strength, when computed on the basis of an undiminished shear modulus G, is not likely to govern. In Fig. 9 we see that if $r_x < r_y$ then FTB will never control; however, Figs. 8 and 10 indicate that minor axis buckling is not always governing. (Compare curves FB(Q)(x - x) and FTB, $G_t = G$ in Fig. 8, and FB(Q)(y - y) and FTB, $G_t = G$ in Fig. 10). There are substantial regions where FTB governs and it must, therefore, be checked for double-angle columns. This check can be made by a simpler method than that recommended by AISC in Ref. 1, e.g.,:

$$P_n = \text{MIN}(P_{ix}, P_i) \tag{13}$$

where P_{ix} is the x - x axis buckling load as determined by the AISC column curve (including a modification for local buckling), and

$$P_i = \left[\frac{P_{iy} + P_{it}}{2H}\right]\left\{1 - \left[1 - \frac{4P_{iy}P_{it}H}{(P_{iy} + P_{it})^2}\right]^{½}\right\} \tag{14}$$

where P_{iy} is the y - y axis buckling load as determined by the AISC column curve (excluding a modification for local buckling), and

$$P_{it} = GJ/r_o^2 \tag{15}$$

LOCAL BUCKLING MODIFICATION

The AISC LRFD Specification accounts for the local buckling of unstiffened slender plate elements by using the Q-factor, where

$$Q = F_{lb}/F_y \tag{16}$$

F_{lb} being the critical local buckling stress which is modified for some inelastic behavior. Q is defined in Appendix B5 in Ref. 1 for several cases, e.g. single-angles, outstanding flanges of W-shapes and stems of T-shapes. Web plate elements are stiffened elements and these are treated by the "effective width method." The AISI Specification for cold-formed steel structures (Ref. 2) has adopted a unified effective width approach which is applied to elements with both stiffened and unstiffened plate elements. Since AISI and AISC have different definitions of the flat width the former starting from the toe of the fillet and the latter using overall dimensions, the values of the coefficients are slightly different. Following are the formulas for the AISC version of the effective width formula:

$$b_e = \rho b \tag{17}$$

where

$$\rho = 1 \text{ for } \lambda_p \le 0.77 \tag{18a}$$

$$\rho = \left(1 - \frac{0.175}{\lambda_p}\right)\left(\frac{1}{\lambda_p}\right) \text{ for } \lambda_p > 0.77 \tag{18b}$$

$$\lambda_p = 1.17\left(\frac{b}{t}\right)\left(\frac{f}{kE}\right)^{½} \tag{19}$$

$$f = F_n \tag{20}$$

where F_n is the critical stress for the column for the whole cross section and $k = 0.425$ is the plate buckling coefficient for an angle. The method of determining the nominal column capacity is as follows:

Step 1: Calculate $\lambda_c = \frac{L}{r\pi}\left(\frac{F_y}{E}\right)^{½}$

Step 2: Determine F_n by Ref. 1, Sec. E2.

$F_n = 0.658^{\lambda_c^2} F_y$ for $\lambda_c \le 1.5$

$F_n = \frac{0.877}{\lambda_c^2}$ for $\lambda_c > 1.5$

Step 3: Calculate b_e using Eq. 17 for each leg of the angle.
Step 4: Determine the effective area A_e
$$A_e = A - [(b_e - b) + (d_e - d)]t$$
where b_e and d_e are the effective widths of the legs with total widths b and d, respectively.
Step 5: Calculate $P_n = A_e F_n$

Figures 11 and 12 are typical plots showing comparisons between the Q-factor and the effective area methods. For unequal-leg angles the Q-factor method gives generally lower strength, while the reverse tends to be the case for equal-leg angle columns. While there is no clear advantage of one over the other approach, the effective area method will unify the treatment of stiffened and unstiffened plate elements.

COMPARISON WITH TESTS

There are two sets of carefully conducted axially loaded recent column tests available, i.e., the tests of John B. Kennedy and Madugula K.S. Murty (Ref. 9) and S. Kitipornchai and H.W. Lee (Ref. 10). Single-angle, double-angle and T-columns with pinned and fixed ends were tested. The histograms of Figs. 13 and 14 show the test-to-prediction ratios for the current AISC-LRFD method (Fig. 13) and the effective area method (Fig. 14).

The probabilistically determined reliability index β variation with the nominal live load-to-dead load ratio is shown in Figs. 15 and 16 for resistance factors $\phi = 0.85$ and 0.90, respectively.

SUMMARY

This paper has shown that the current (1991) method of designing angle columns by the AISC LRFD Specification is conservative and more complicated than necessary. A simpler and less conservative method of angle-column design was presented.

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NOMENCLATURE

τ	–	tangent modulus ratio
ν	–	Poisson's ratio
A	–	cross sectional area
A_e	–	effective area
b_e	–	effective width
C_w	–	warping constant
D	–	angle leg dimension
E	–	modulus of elasticity
F_n	–	buckling stress
F_y	–	yield stress
G	–	shear modulus
H	–	$1 - u_o^2/r_o^2$
I_z, I_u	–	moments of inertia about the z and μ axis
J	–	torsion constant
k	–	plate-buckling coefficient
L	–	column length
P_n	–	nominal column capacity
r_o^2	–	$u_o^2 + z_o^2 + (I_u + I_z)/A$
r_z, r_u	–	radii of gyration
t	–	angle thickness
u_o, z_o	–	shear center coordinates

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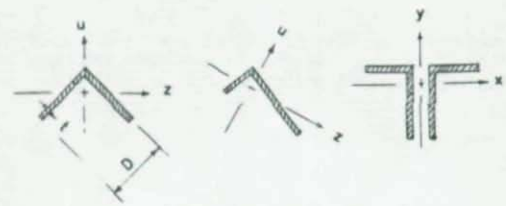
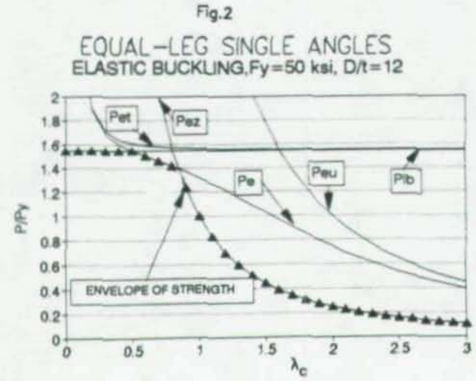


Fig.1 Angle Configurations

Fig.2



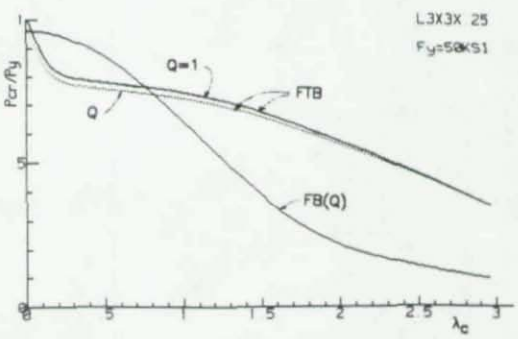


Fig.3 AISC Angle–Column Design Criteria, LRFD Specs. APPENDIX E3

FTB: Flexural–Torsional Buckling

Q: FTB with Q–factor

Q=1: FTB without Q–factor

FB(Q): Flexural Buckling with Q–factor

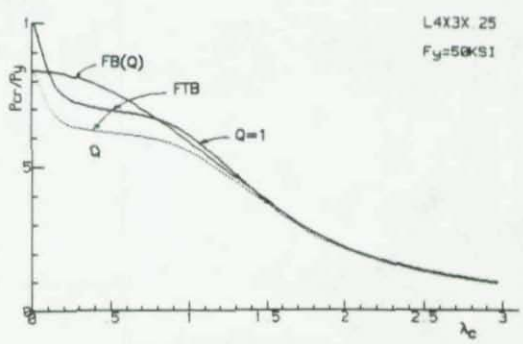


Fig4. AISC LRFD Criteria

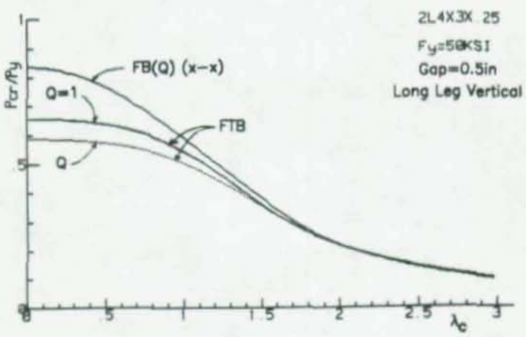


Fig.5 AISC LRFD Criteria

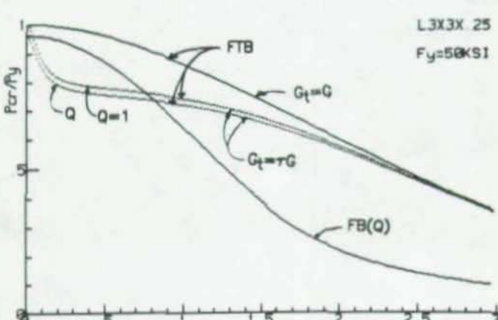


Fig.6 AISC LRFD Citeria Compared to Tangent Modulus Solution

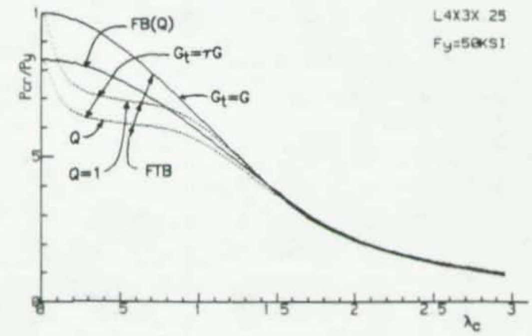


Fig.7 AISC Compared to Tangent Modulus

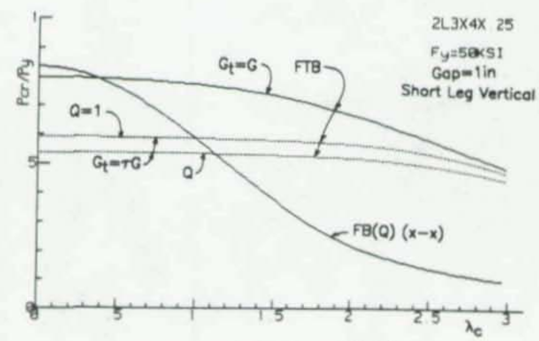


Fig.8 AISC Compared to Tangent Modulus

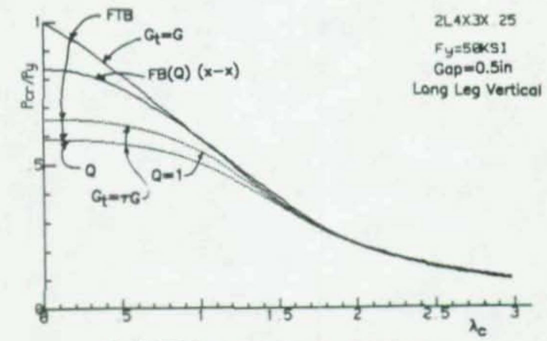


Fig.9 AISC Compared to Tangent Modulus

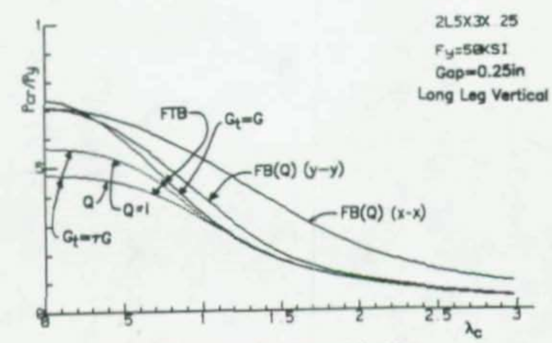


Fig.10 AISC Compared to Tangent Modulus

Fig.11

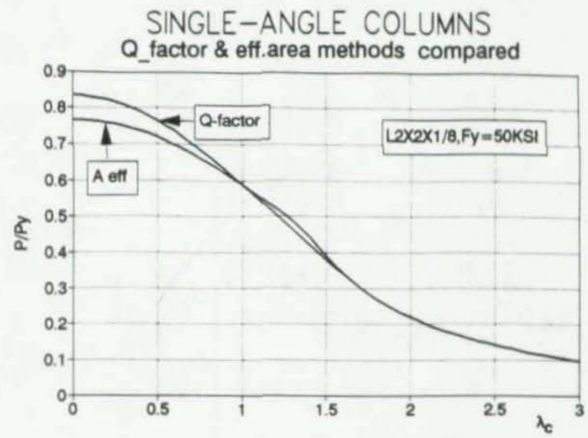

SINGLE-ANGLE COLUMNS
Q_factor & eff.area methods compared
Q-factor
A eff
L2X2X1/8,Fy=50KSI
P/Py
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.5
1
1.5
2
2.5
3
λc

Fig.12

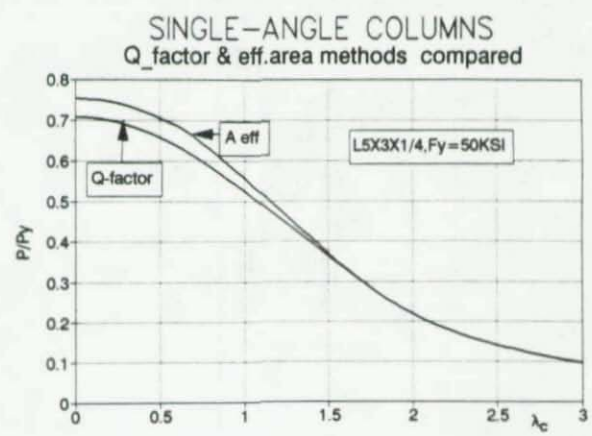

SINGLE-ANGLE COLUMNS
Q_factor & eff.area methods compared
A eff
Q-factor
L5X3X1/4,Fy=50KSI
P/Py
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.5
1
1.5
2
2.5
3
λc

FIG. 13
TEST/PREDICTION
AISC-LRFD 1986

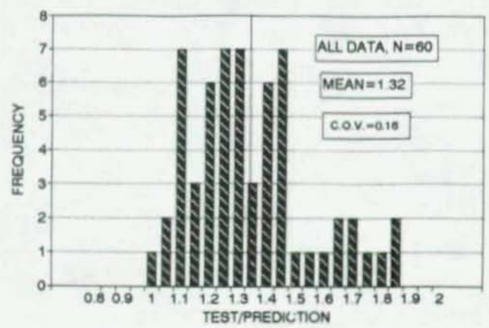


FIG. 14
TEST/PREDICTION
PROPOSED METHOD

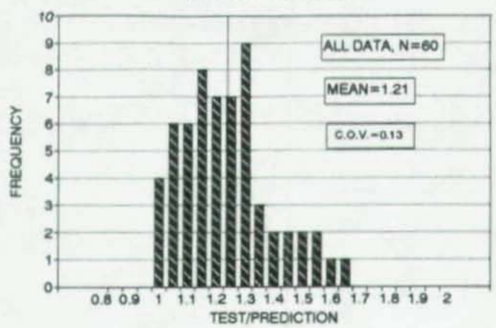


FIG. 15

BETA FOR ANGLE COLUMNS

PHI=0.85

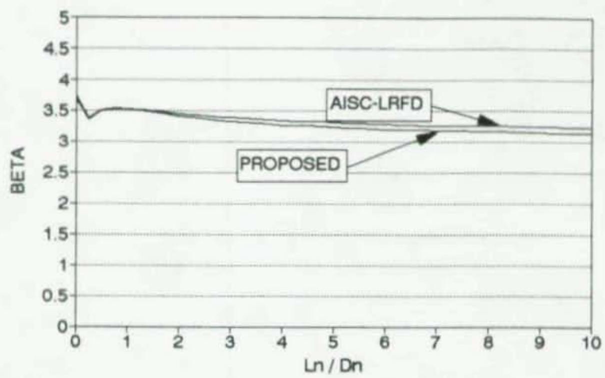


FIG. 16

BETA FOR ANGLE COLUMNS

PHI=0.90