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Cylindrical Shell Buckling Strength according to the "Overall Method" of Eurocode 3 - Background and Applicability to the Design of High Strength Steel Circular Hollow Sections

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Abstract

With the introduction of the Eurocodes, a new design philosophy for the design of cylindrical shells against buckling was made available to structural engineers in Europe: the "Overall Method", which makes use of the so-called "Overall Slenderness" of the shell to determine the appropriate (local) buckling reduction factor. There is a certain similarity between this concept and the Direct Strength Method (DSM) used in North America for the design of thin-walled cross-sections: the "Overall Method" also makes use of the results of (numerical) linear buckling analyses (LBA) for the whole shell to determine the slenderness and consequently the buckling reduction factor. In the case of the "Overall Method", the combined load case, which may lead to shear, hoop and axial stresses, is thereby considered in the LBA. The paper discusses the background and application of this method and complications arising in certain cases. In particular, the application of the Eurocode rules to the design of slender ("class 4") cylindrical hollow sections (CHS) will be discussed. While generally suitable for the design of large diameter cylindrical shells (tanks, silos), the application of the rules to the design of locally slender CHS requires a significant amount of interpretation by the code user and is generally not very accurate. Due to the increased use of (very) high strength steel grades for hollow sections in Europe (with yield strengths of f_y=770 MPa and beyond), the local buckling design of HS steel CHS is becoming increasingly relevant. First steps by the authors towards the development of DSM-like design rules for high strength steel CHS under combined load cases will be discussed.

1. Eurocode 3 – Part 1-6 and the "Overall" or "MNA/LBA" Method for Buckling Design

1.1 Eurocode 3 – Part 1-6

Within the Eurocode suite of design standards, Eurocode 3 is dedicated to the wide application range of steel structures. Eurocode 3 - i.e. the CEN standards with the numbering "EN 1993" - is subdivided into twenty individual documents ("parts"), which deal with the structural design of different types components or structural assemblies. The design of (cylindrical) shell structures is regulated in general terms in EN 1993 part 1-6 (CEN 2010), as well as more specifically in several application standards that deal with the design of chimneys, silos, tanks, etc.

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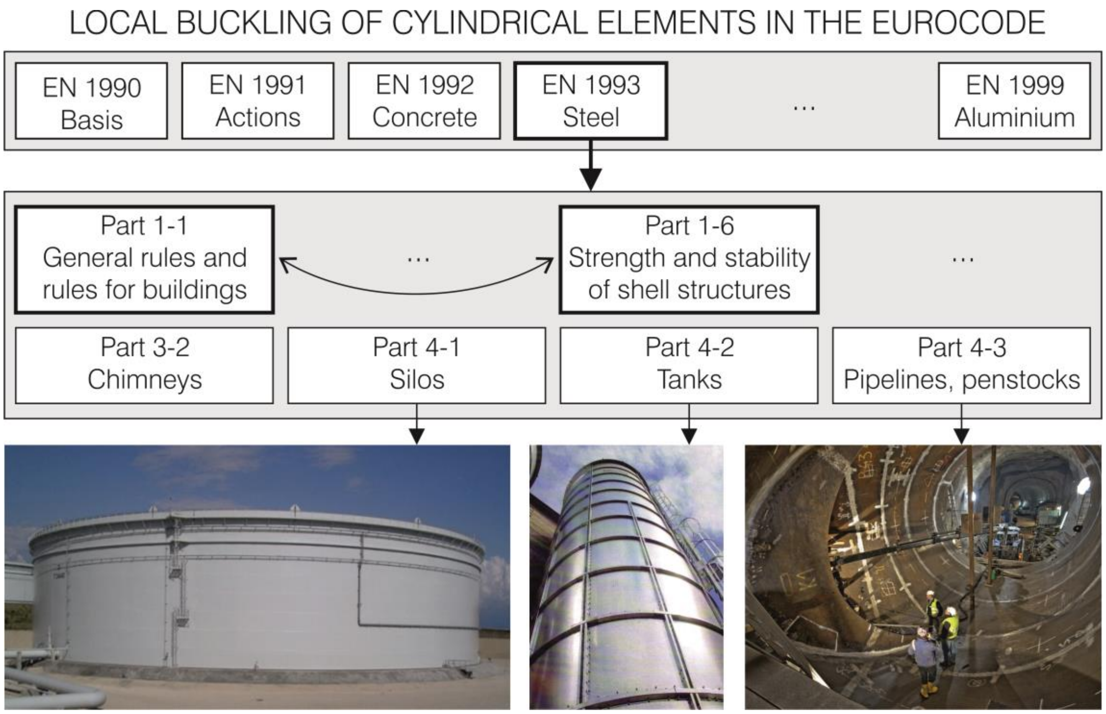


Figure 1: Design of cylindrical steel shells against buckling in the Eurocode

Fig. 1 summarizes the structure of the Eurocode and the parts relevant for the design of cylindrical shells. Some typical applications of part 1-6 are also shown in the figure.

In order to facilitate the understanding of the variables used in this paper, the notation commonly used in the Eurocode to designate stress components and orientations is summarized in Fig. 2, whereby the focus was put on the axial stress components σ_x, which will be dealt with in detail in the following sections.

Due to the high slenderness (in terms of diameter to thickness ratios D/t) of the common cylindrical structures shown in Fig. 1, the design against buckling takes a central role in EN 1993-1-6. A variety of methods are offered in the code, which account for the differences in available design tools, time and expertise in typical design offices:

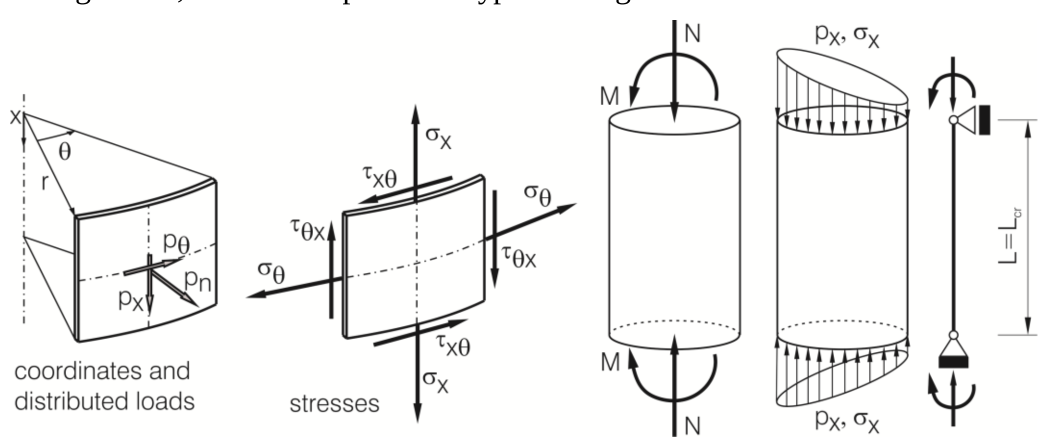


Figure 2: Notation for stress components and actions.

i. The traditional method for the design against buckling of cylindrical shells subjected to a general load case (causing meridional, circumferential and/or shear stresses) requires the calculation of the elastic critical shell buckling stresses and of appropriate shell buckling knockdown factors for the individual stress components. Appropriate “hand formulae” from the literature - or the Annex of the code itself - can be used for the calculation of these quantities. The interaction between the different stress components is dealt with through an interaction formulation.

ii. The increase of availability and expertise in the use of advanced numerical analysis techniques and software packages was acknowledged in the code by the introduction of the so-called “MNA/LBA Method” or “Overall Method”. This approach is found in section 8.6 of EN 1993-1-6 - “*Design by global numerical analysis using MNA and LBA analyses*”. The acronyms used represent a **M**aterially **N**onlinear **A**nalysis, used to determine the plastic limit load (ignoring instability), and a **L**inear **B**uckling **A**nalysis, used to calculate the linear elastic bifurcation load of the shell (ignoring imperfections and plasticity). The name “Overall Method” stems from the use of an “overall” slenderness, valid for the whole structure and the studied loading condition, and calculated on the basis of the global MNA and LBA analyses. The basic principles of the method are summarized in Fig. 3 and are described in section 2.2.

iii. Finally, it shall be mentioned that EN1993-1-6 also contains an additional, even more advanced method, which makes use of analysis techniques that are **G**eometrically and **M**aterially **N**onlinear and account for **I**mperfections – GMNIA. This represents a full elasto-plastic finite displacement numerical (FEM) analysis with appropriate geometric and structural imperfection definitions. The method is used in later sections of this paper as a research tool. For the design of typical cylindrical shells against buckling, the code recommends great care and requires validation effort by the structural engineer who wishes to employ it. Contrary to the “traditional” and the “Overall – MNA/LBA” approaches, it is not generally seen as a design tool, but rather as a tool to verify existing structures in certain cases and applications, as well as a method suitable for code-making and applied research.

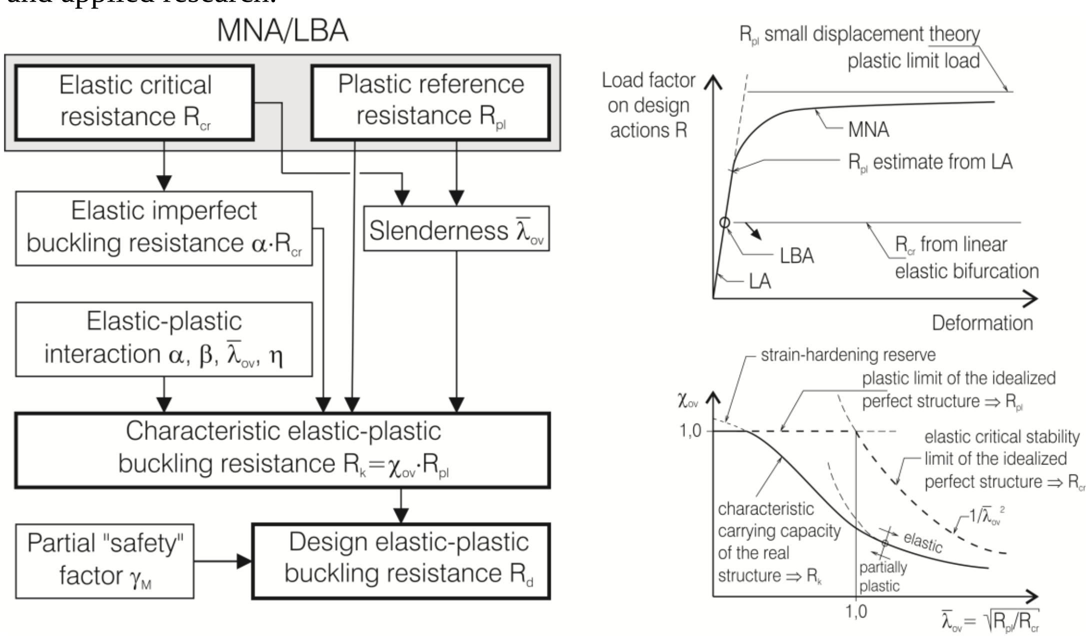


Figure 3: The “Overall” or “MNA/LBA Method” in EN 1993-1-6

1.2 Features of the "Overall – MNA/LBA Method"

The introduction of the "Overall Method" into the Eurocode can be ascribed to the work of the members of the Technical Committee 8 – Working Group 8.4 of the European Convention for Constructional Steelwork (ECCS). The basic principles of the method, among other aspects of EN 1993-1-6 are described in detail in an ECCS recommendation (Rotter & Schmidt 2008) and related papers (Rotter 2011), and are summarized as follows:

i. The method considers a combined load case and operates with load amplification factors to describe the different amplitudes and definitions of load levels and corresponding resistances. Following the notation of the ECCS recommendation, "R" is used for these load factors.

ii. The (geometrically linear) plastic collapse load "R_{pl}" for the studied load case is determined using an appropriate "MNA" numerical analysis. This load level is thought of as an upper bound of the shell's real load-carrying capacity.

iii. A linear buckling analysis (LBA) is performed in order to determine the first, critical bifurcation load "R_{cr}" of the shell under the studied load case. This bifurcation load is thought to be representative for the buckling sensitivity of the shell.

iv. The two quantities R_{pl} and R_{cr} are used to determine the "overall" slenderness $\bar{\lambda}_{ov}$:

$$\bar{\lambda}_{ov} = \sqrt{\frac{R_{pl}}{R_{cr}}} \quad \left(\cong \sqrt{\frac{f_y}{\sigma_{crit}}} \right) \tag{1}$$

v. An "appropriate" shell buckling knockdown factor is then determined. The Eurocode uses the Greek letter "χ" for many buckling reduction factors – including shell buckling and beam-column failure modes. In EC3-1-6, the factor "χ_{ov}" is a function of the slenderness $\bar{\lambda}_{ov}$ and of the parameters α, β and η, which describe the shape of the reduction curve. In correspondence with the three main ranges of buckling response ("plastic", "elastic-plastic", "elastic"), the formula for χ_{ov} is represented as follows:

$$\chi_{ov} = 1,0 \qquad \text{when } \bar{\lambda}_{ov} \leq \bar{\lambda}_{ov,0} \tag{2a}$$

$$\chi_{ov} = 1 - \beta \left[\frac{\bar{\lambda}_{ov} - \bar{\lambda}_{ov,0}}{\bar{\lambda}_{ov,p} - \bar{\lambda}_{ov,0}} \right]^{\eta} \qquad \text{when } \bar{\lambda}_{ov,0} < \bar{\lambda}_{ov} < \bar{\lambda}_{ov,p} \tag{2b}$$

$$\chi_{ov} = \frac{\alpha}{\bar{\lambda}_{ov}^{2}} \qquad \text{when } \bar{\lambda}_{ov,p} < \bar{\lambda}_{ov} \tag{2c}$$

vi. The expressions 2a-2c are equivalent to the formulae used in EN 1993-1-6 for the individual buckling cases (meridional compression, circumferential compression, shear) in the "traditional" method mentioned in section 1.1. For these basic cases, the coefficients to be used are given in Tables 1 and 2, as well as by Eq. 3 for the case of meridional compression. As can be seen in Table 2, the coefficients for the basic cases are dependent on "Quality Classes" A to C, which are representative of different ranges of expected geometric imperfection amplitudes. The impact of the quality classes on the resulting reduction factor is shown in Fig. 4a for the case of constant meridional compression ($\chi_{x,N}$, index "N" to represent a global normal force for the cylinder).

vii. The choice of "appropriate" coefficients to be used in the context of the "Overall Method" is left to code user. In practice, this means that a rather large "range" of possible reduction factors exists. This is represented in the shaded are in Fig. 4a.

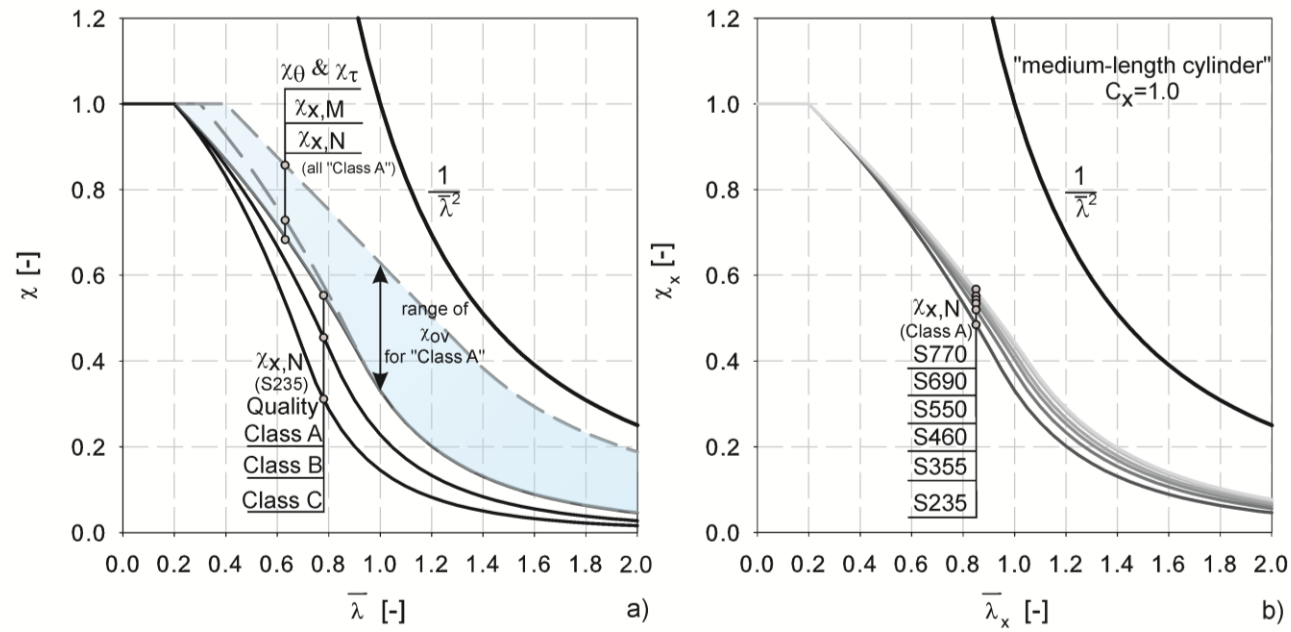


Figure 4: Range of buckling reduction factors $\chi=\chi_{ov}$ (a); influence of the steel grade on the reduction factor χ_x for meridional compression.

Table 1: Coefficients for Eq. 2a-2c for basic buckling cases

Buckling case	$\bar{\lambda}_{ov,0}$	α	β	η
meridional compression σ_x	0.2[1]	Eq. 3 + Table 2		
circumferential compression σ_θ	0.4	Table 2	0.60	1.00
shear $\tau_{x\theta}$	0.4	Table 2		

1. Can be set to 0.3 for cylinders in bending when certain conditions are met.

for meridional compression:

$$\alpha_x = \frac{0.62}{1+1.91\cdot\left(\Delta w_k / t\right)^{1.44}} \tag{3a}$$

with

$$\Delta w_k / t = \frac{1}{Q}\cdot\sqrt{\frac{r}{t}} \tag{3b}$$

Table 2: Coefficients dependent on the “Quality Classes”

Buckling case	Class A	Class B	Class C
meridional compression σ_x	Q=40	Q=25	Q=16
circumferential compression σ_θ	α=0.75	α=0.65	α=0.50
shear $\tau_{x\theta}$	α=0.75	α=0.65	α=0.50

The ECCS recommendation points out that meridional compression is particularly critical and therefore recommends using the coefficients for this loading case whenever meridional stresses are significant.

viii. The case of meridional compression is thus of particular relevance for the application of the method – as well as for the application further studied in this paper, circular hollow sections loaded in compression and/or bending. The reduction factors $\chi_{x,N}$ for the quality class A are plotted again in Fig. 4b, for steel grades ranging from conventional mild steel (S235) to high-strength S770 steel. As can be seen in the figure, Eq. 2 and 3 lead to an influence of the steel grade (the European steel grade notation is used, with "S" for structural steel and the figure behind it representing the nominal yield stress f_y in MPa). This is due to the – mechanically coherent – definition of the geometric imperfection factors α and w_k/t as function only of the cylinder's geometry (square root of r/t), while the normalized slenderness $\bar{\lambda}$ is also a function of the yield stress, see Eq. 1. With this definition, the buckling knockdown factor χ_x is higher for a higher yield stress value and a certain fixed value of the normalized slenderness $\bar{\lambda}$.

2. Significance of the Elastic Buckling Stress and Analogies with the "Direct Strength Method"

Quite fundamentally, some of the features and principles of the "Overall Method" described above should be recognizable to North American engineers familiar with the "Direct Strength Method" (DSM) for the design of cold-formed steel (Schafer 2008). In particular, both approaches make use of the results of sophisticated (often, but not necessarily numerical) linear buckling analyses and reduce the small-displacement plastic/yield load by different buckling knockdown factors, which in turn depend on the underlying elastic buckling mode. The quality and interpretation of the LBA result is thus of fundamental importance to both methods.

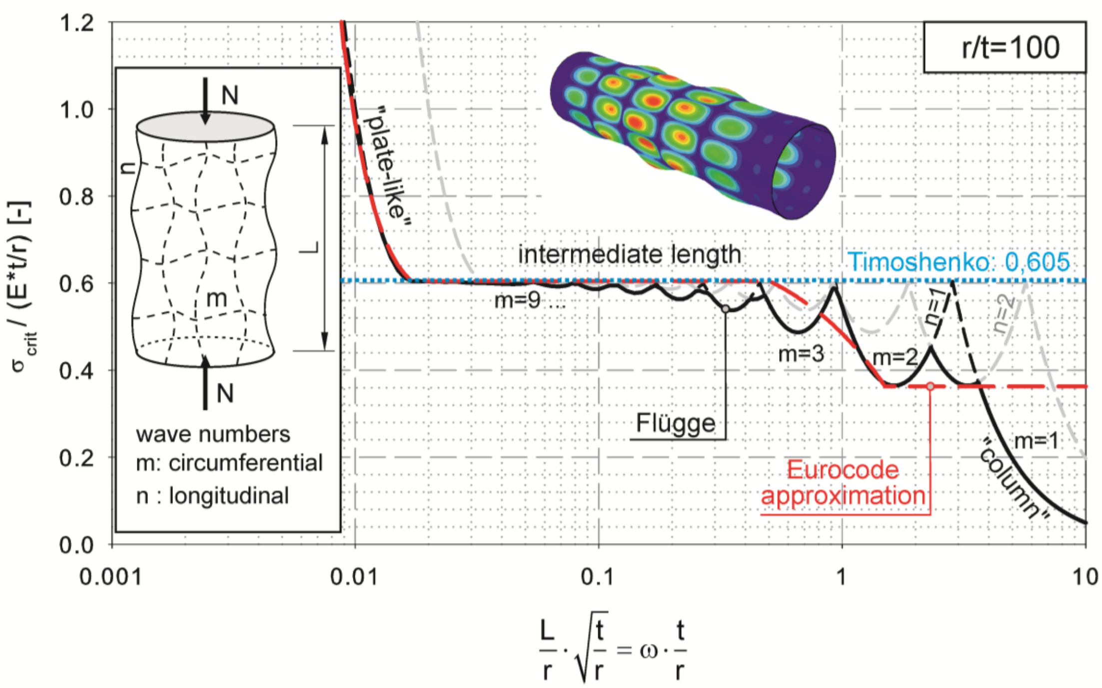


Figure 5: Theoretical expressions for the critical buckling stress of a cylinder under meridional compression – analogy with the DSM "signature curve".

In the DSM literature, the concept of a "signature curve" is used, i.e. a representation of the critical buckling stresses of a certain cross-sectional shape as function of the buckling half-wave length.
In the shell buckling literature, a similar concept is used, see Fig. 5. This figure shows a number of theoretical solutions for the critical elastic buckling stress of a cylinder with r/t=100, plotted over the dimensionless parameter $\omega \cdot t / r$, with:

$$\omega = \frac{L}{\sqrt{r \cdot t}} \tag{4}$$

The case shown in Fig. 5 is the "classical" case of constant axial compression and "hinged" end boundary conditions. The best known and simplest solution to this problem is already mentioned by Timoshenko (1934), and relates the elastic buckling stress to the modulus of elasticity E and the thickness to radius ratio t/r:

$$\sigma_{crit} = 0.605 \cdot E \cdot \frac{t}{r} \tag{5}$$

Due to the significance and simplicity of this formula, it is customary to represent the actual circumferential elastic buckling stress of a studied case as ratio of σ_{crit}/(E*t/r); in cases where Equ. 5 is correct, this leads to a ratio of 0,605 in this type of representation.
A more rigorous and general solution to the elastic buckling problem was presented by Flügge (1974) – among others – and is also plotted in Fig. 5. Again in analogy with the "signature curve" of the DSM, the representation of the "exact" solution of Flügge over the parameter ω*t/r allows one to discern three very distinct buckling modes, which depend on the circumferential and longitudinal wave numbers m and n, respectively:

i. for very short cylinders, a "plate-like" buckling mode occurs, where the very shallow cylinder essentially behaves like a short flat plate in compression.
ii. in the intermediate length range, a multitude of buckling modes with (almost) identical critical stresses exist: axisymmetric and chequerboard modes (see also Fig. 11) with varying wave number m and n.
iii. Finally, with increasing length the number of unique, lowest local buckling modes decreases, along with the value of σ_{crit} itself. Eventually, the column buckling stress is reached, which manifests itself as a buckling mode with n=m=1 in the studied pin-ended case.

The equations of Flügge are cumbersome to apply and only valid for certain boundary and loading conditions. Approximations to the "exact" solution for different loading conditions were developed by various authors. EN 1993-1-6 contains one set of approximations of this type, see the red curve in Fig. 5. While the reader is referred to the code for the exact formulae, the general format is as given in Equ. (6):

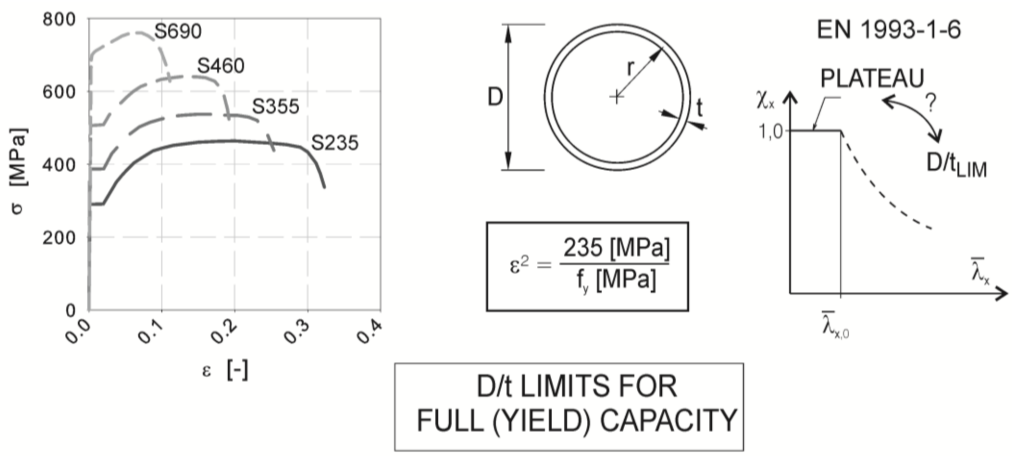
$$\sigma_{crit} = 0.605 \cdot C_x \cdot E \cdot \frac{t}{r} \tag{6}$$

Eq. 6 is identical to Eq. 5, with the exception of the factor C_x. This factor varies from 0.6 to 1.0 for long to medium-length cylinders. Interestingly, the expression provided by EN 1993-1-6 does not represent a lower bound of the "exact" solution by Flügge.

3. High-Strength Steel Cylindrical Hollow Sections – "Slender" Cylinders in Compression?

3.1 General considerations

The classical fields of application of the shell buckling rules of EN 1993-1-6 were already pointed out in section 1, Fig. 1 – i.e. tanks, silos, large-diameter masts and penstocks, etc. However, a new field of application of the shell buckling rules may become of relevance: the determination of the local buckling strength of circular hollow sections (CHS) made of high-strength (HS) steel. In Europe, CHS are produced as hot-finished or cold-formed welded sections in accordance with the product standards EN 10210 and EN 10219 (CEN 2006a, 2006b), respectively. With few exceptions, commonly used diameters do not exceed 600mm in the case of hot-finished and 1000mm in the case of welded sections produced to these standards; the vast majority of employed sections are far smaller, and typical diameter-to-thickness D/t ratios stay well below 100. CHS made of mild steel grades are usually designed in accordance with design standards such as the general part 1-1 of EN 1993 (CEN 2005), the AISC (2010) specification for steel buildings, CIDECT (2010) design guidelines, or – until very recently - national European standards published e.g. by BSI (2000) or DIN (2008). These documents contain limiting values for D/t – see Fig. 6-, below which the CHS can be considered to be (at least) "semi-compact", i.e. the yield stress/ elastic capacity can be reached without local buckling.



EUROCODE - GENERAL PART (EN 1993-1-1) : $D/t \le 90\,\varepsilon^2$ ...compression & bending

BRITISH STANDARD (BS ...) : $D/t \le 94\,\varepsilon^2$ ...compression
$D/t \le 164\,\varepsilon^2$ ...bending

CIDECT GUIDELINES : $D/t \le 90\,\varepsilon^2$ ...compression & bending

GERMAN STANDARD DIN 18800-1 : $D/t \le 70\,\varepsilon^2$ ...compression
$D/t \le 90\,\varepsilon^2$ ...bending

AISC : $D/t \le 0.11\,E/f_y \mathrel{\hat{=}} 98\,\varepsilon^2$ ...compression
$D/t \le 0.31\,E/f_y \mathrel{\hat{=}} 277\,\varepsilon^2$ ...bending

PLATEAU EN1993-1-6 - SHELL BUCKLING :

$$\bar{\lambda}_x = \sqrt{\frac{f_y}{0.605 \cdot \underbrace{C_x}_{0.6\,...\,1.0} \cdot E \cdot t/r}} \overset{!}{=} \bar{\lambda}_{x,0} = \begin{matrix} 0.2 \text{ (compr.)} \\ 0.3 \text{ (bending)} \end{matrix} \xrightarrow{r \approx D/2} \begin{matrix} D/t \le 26\,\varepsilon^2 \text{ - } 43\,\varepsilon^2 & \text{... compression} \\ D/t \le 97\,\varepsilon^2 & \text{... bending} \end{matrix}$$

Figure 6: D/t limits for full (yield) capacity – limit between semi-compact and slender sections.

Table 3: D/t limits for non-compact sections acc. to different design standards and different European steel grades.

Steel Grade [f_y in MPa]	ε^2 $=235/f_y$ (f_y in MPa)	EN 1993-1-1	BS 5950:2000		DIN 18800-1		AISC (2010)		EN 1993-1-6	
			compr.	bending	compr.	bending	compr.	bending	compr. [1]	bending [2]
S235	1.00	90.00	94.00	164.00	70.00	90.00	98.00	277.00	43.00	97.00
S355	0.66	59.58	62.23	108.56	46.34	59.58	64.87	183.37	28.46	64.21
S460	0.51	45.98	48.02	83.78	35.76	45.98	50.07	141.51	21.97	49.55
S550	0.43	38.45	40.16	70.07	29.91	38.45	41.87	118.35	18.37	41.45
S690	0.34	30.65	32.01	55.86	23.84	30.65	33.38	94.34	14.64	33.04
S770	0.31	27.47	28.69	50.05	21.36	27.47	29.91	84.54	13.12	29.60
S890	0.26	23.76	24.82	43.30	18.48	23.76	25.88	73.14	11.35	25.61

1. For "medium-length" cylinders in compression, C_x=1,0; 2. For bending: C_x=1,0, $\lambda_{x,0}$=0.3

Whenever necessary (e.g. in the case of the AISC limits), the D/t ratios in Fig. 6 were re-calculated on the basis of a value of the Young's Modulus E equal to 210 GPa, as specified in the Eurocode. As can be seen in the figure, the ratio $\varepsilon^2=f_y/235$ between the standardized yield stress (in MPa) of the common mild steel in Europe (S235) and of the used material enters the definition of the limiting value of D/t. This is explicable in light of the definition of slenderness used in Eq. 1 and the formula for σ_{crit} in Eq. 6: with a given "plateau" value of the buckling knockdown factor curve ($\bar{\lambda}_{x,0}$), the corresponding D/t (~2 r /t) value can be calculated by solving the expression at the bottom of Fig. 6 for r/t ~ (D/2)/t.

Contrary to the case of mild steel CHS, for HS steel the resulting limit values of D/t are rather low, see Table 3. This means that many more sections will fall into the slender category according to these common rules. Furthermore, the HS steels on the European market typically feature a stress-strain (σ-ε) relationship as sketched on the top left corner of Fig. 6, with comparatively little "over-strength" – in terms of yield stress- compared to the specified minimum value, low strain hardening (difference between tensile strength and yield stress), and relatively low elongation at necking ε_u. This means that the background to the D/t limits themselves might be called into question when applied to (very) high-strength steel grades.

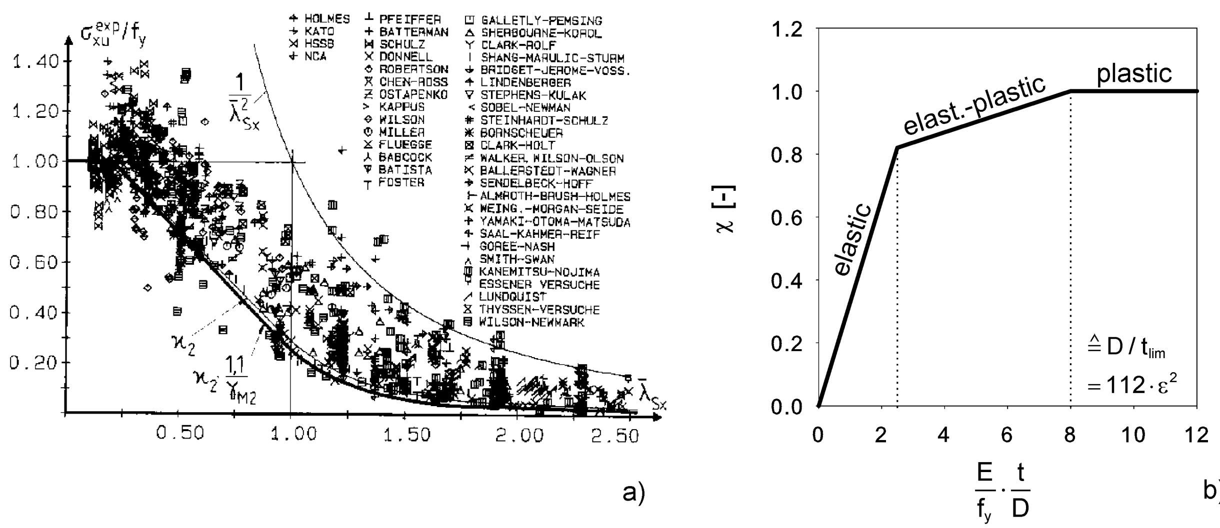


Figure 7: Background tests for the buckling knockdown factor χ_x in EN 1993-1-1 (a), Plantema diagram as background for the D/t limits for semi-compact CHS in international building codes (b)

The known background to the limits in Fig. 6 and – equivalently – to the "plateau" value of the buckling reduction factor χ_x in EN 1993-1-6 is shown in Fig. 7. Thereby, Fig. 7a is taken from the commentary to DIN 18800 (Lindner et al. 1994) and shows an extensive collection of 1200 international buckling tests for meridional compression of metallic cylinders; these tests are compared to the κ_2 curve of DIN 18800-4, which is quite similar to the current $\chi_{x,N}$ curve for quality class "B" in EN 1993-1-6 (compare Fig. 4a). The huge scatter – and work undertaken to reduce it – is commented upon in the mentioned reference. Nevertheless, the plateau value of $\bar{\lambda}_{x,0}=0.2$ is stated to be appropriate. It shall be noted that some higher-strength steel grades are included in the plot, as well as some non-ferrous metallic alloys.
The original source / function that serves as justification for the limit values of ca. 90-100 D/t ε^2 mentioned in most steel design codes is shown in Fig. 7b: the shown relationships were first given in the work by Plantema (1946) and are based on a series of original tests on mild steel tubes.

3.2 Buckling knockdown values in various standards

Once a studied (HS steel) CHS is determined to fall into the "slender" cross-sectional range, it becomes necessary to account for local buckling in the design formulations. Eurocode 3 (general part 1-1) contemplates this case only in passing, as it refers the user to EN 1993-1-6 and the shell buckling rules therein; it shall be noted that this is quite impractical and often not fully consistent with EC3 part 1-1, as the latter code makes use of "effective cross-sections" (effective area A_{eff} and section modulus W_{eff}, reduced from the gross area A and the – elastic - section modulus W_{elast}) when dealing with slender ("class 4" in EC3) sections, yet EN 1993-1-6 contains no rules for the determination of effective sections.
Other design codes contain more easily applicable rules for either buckling knockdown factors or effective areas: For the studied cases of isolated compression and bending, both can easily be rewritten in terms of the reduction factor χ_x. For example, in BS5950-1 (BSI 2000), the following two expressions are given (rewritten here to be based on S235 steel instead of the original S275):

$$\chi_{x,N} \triangleq \frac{A_{eff}}{A} = \left[\frac{94}{D/t}\cdot\varepsilon^2\right]^{0.5} \quad (7)$$

$$\chi_{x,M} \triangleq \frac{W_{eff}}{W_{elast}} = \left[\frac{164}{D/t}\cdot\varepsilon^2\right]^{0.25} \quad (8)$$

Similarly, the AISC (2010) specification contains the following rules (in section E and F):

$$\chi_{x,N} \triangleq Q = \frac{0.038\cdot E}{f_y\cdot(D/t)} + \frac{2}{3} \cong \frac{34}{D/t}\cdot\varepsilon^2 + 2/3 \quad (9)$$

$$\chi_{x,M} \triangleq \frac{M_{n,AISC}}{f_y\cdot W_{elast}} = \begin{cases} \alpha_{plast} = \frac{4}{\pi}\cdot\left(1+\frac{t}{D}\right) & \text{when } D/t \le 62.5\cdot\varepsilon^2 \quad (10a) \\ \left(1+\frac{0.021\cdot E}{f_y\cdot D/t}\right) & \text{when } D/t \le 277\cdot\varepsilon^2 \quad (10b) \\ \left(\frac{0.33\cdot E}{f_y\cdot D/t}\right) & \text{when } D/t > 277\cdot\varepsilon^2 \quad (10c) \end{cases}$$

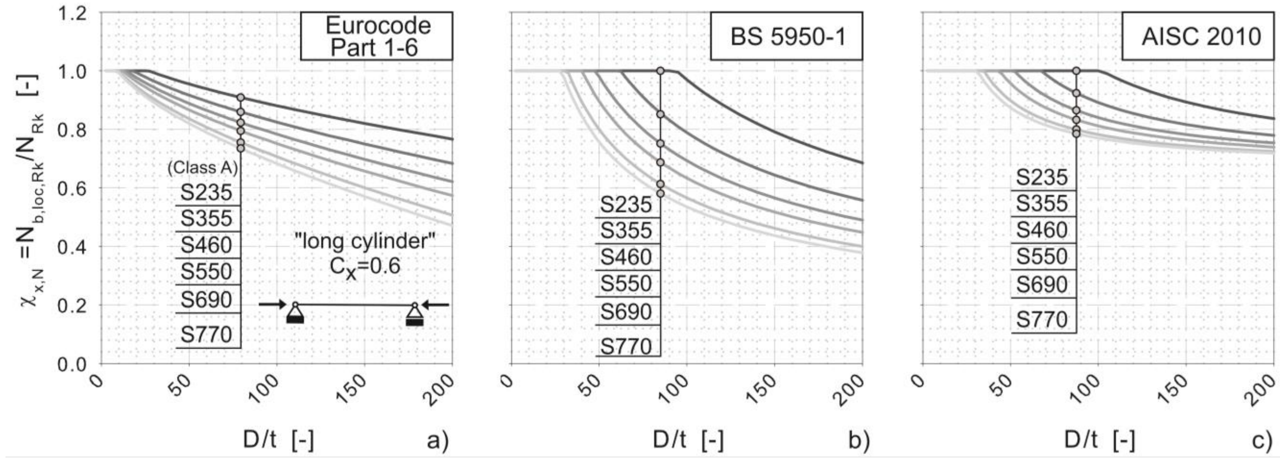


Figure 8: Buckling knockdown factors for CHS according to various standards – *CHS in axial compression*

In Fig. 8 and Fig. 9, the equations 7 to 10 are compared to the EN 1993-1-6 values of $\chi_{x,N}$ and $\chi_{x,M}$ for axial compression and bending, respectively. Thereby, all considerations of safety (safety factors γ_M or ϕ in the different codes) are omitted, leading to the comparison of nominal "characteristic" strength values.

It is evident that there are very large differences between the predicted (local) buckling strengths according to the different design standards. The following point can be summarized:

i. Particularly for isolated compression (value $\chi_{x,N}$), the "plateau" value of D/t, for which the full capacity $A^* f_y$ may be considered in design, is much lower according to EC3-1-6. In fact, the onset of the local buckling reduction factors is far lower than the limit of 90 ε^2 in EC3 part 1-1, see also Fig. 6.

ii. However, for higher-strength steel grades the EC3-1-6 reduction factors for compression can be noticeably larger than e.g. the values of BS5950-1, especially for higher D/t values. The AISC rules, on the other hand, consistently stay above the EC3 shell buckling values, and feature a peculiar horizontal asymptote at $\chi_{x,N}$=2/3; this is self-evident from Eq. 9, yet mechanically rather nonsensical.

iii. For bending, the EC3 and BS5950-1 rules show the typical discontinuity of strength in the semi-compact range between compact ("class 2" in EC3) and slender ("class 4")

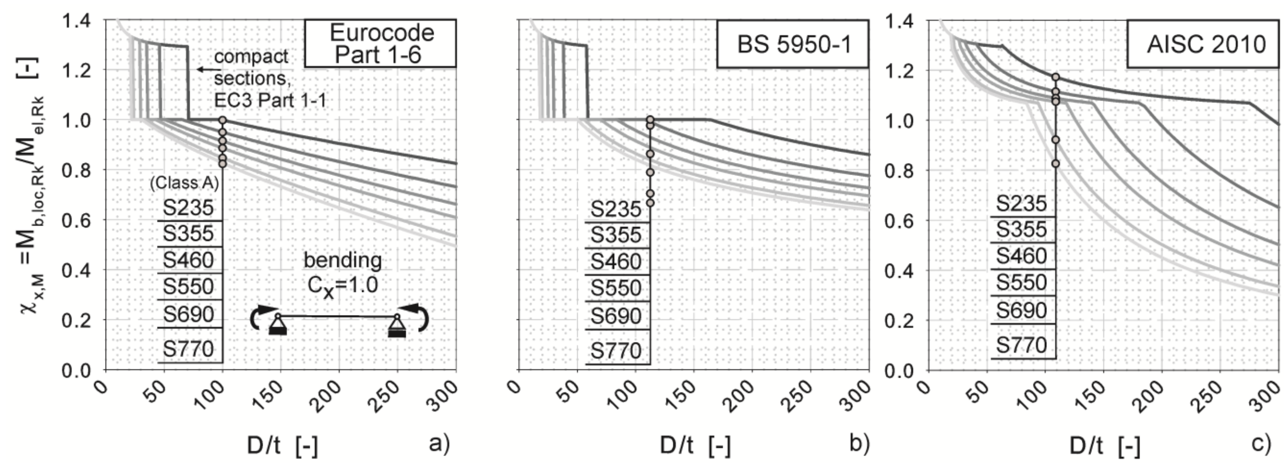


Figure 9: Buckling knockdown factors for CHS according to various standards – *CHS in bending*

cross-sections: according to both codes, the plastic capacity can be used for compact sections, while semi-compact sections can only be designed up to first yield. (Note that this particular issue was already addressed in (Semi-Comp 2007), albeit not for CHS). Aside from this discontinuity, both codes predict lower reduction factors for bending than for compression for the same D/t value, which is mechanically coherent. The AISC specification, on the other hand, has the advantage of containing an explicit strength transition for semi-compact sections, while the reduction factors for higher-strength steel grades are much lower than for the compression case; again, this second point is not mechanically logical.

iv. Quite generally, there is very little consistency between the predicted buckling knockdown factors, particularly for HS steel, with the AISC and BS 5950-1 rules being sometimes "much safer", other times "unsafe" when compared to the EN 1993-1-6 rules. In light of the test evidence shown in Fig. 7a, this can be considered to be problematic.

4. HS steel CHS in the "STSS" Research Framework – Projects "Hollopoc" and "Hollosstab"

4.1 General remarks

The situation concerning the design for (local) buckling of (HS steel) CHS discussed in section 3 clearly underpins the need for further research in this area: evidently, the existing design rules are not directly applicable to this case and/or far exceed their original scope of validation.

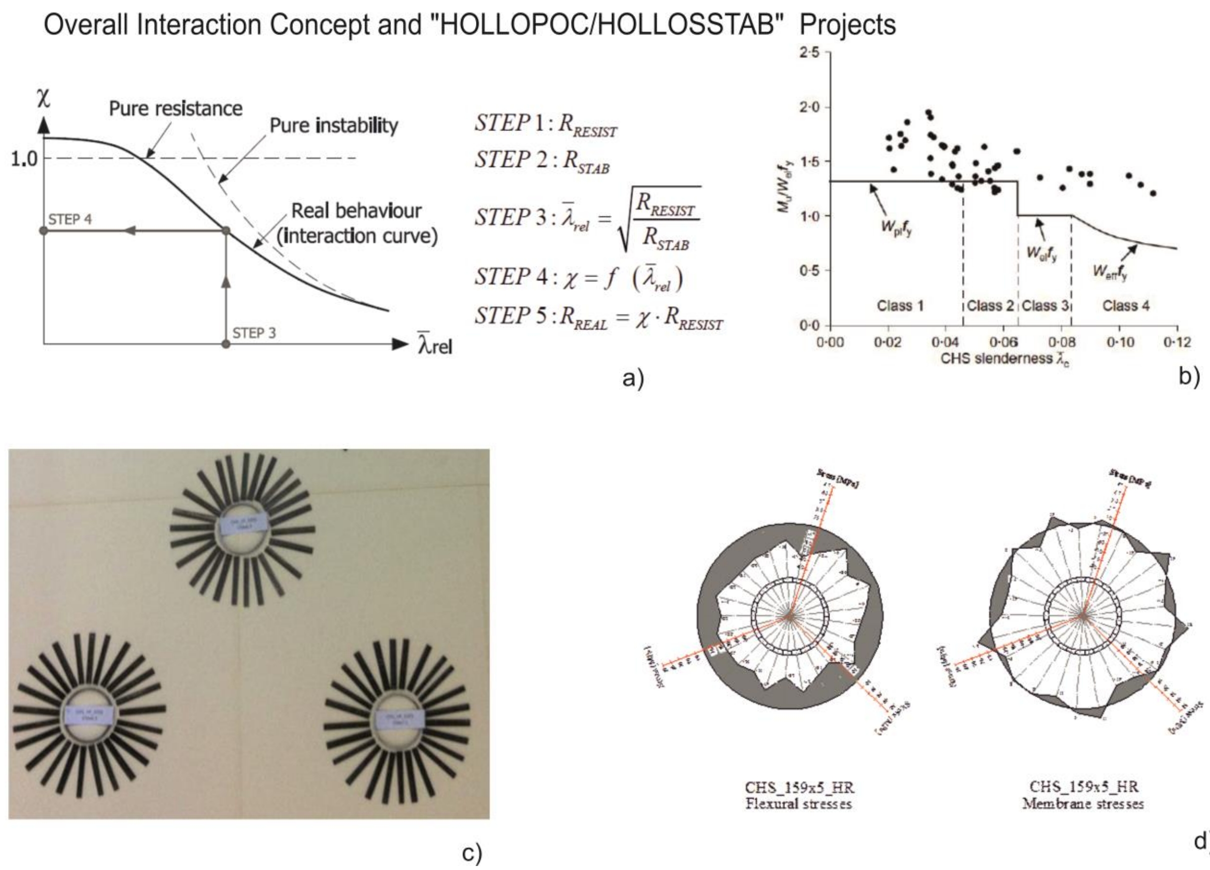


Figure 10: "Hollopoc" / "Hollosstab" projects – OIC concept (a) and some preliminary results: previous bending tests on CHS carried out at Imperial College London (b), residual stress measurements at HES Fribourg (c) and (d).

While section 3 was solely concerned with "local buckling", it is obvious that CHS elements are usually (also) globally slender and thus potentially susceptible to flexural column buckling. Very little evidence has been gathered so far on the interactive local/global buckling of HS steel cylindrical elements; an extrapolation from findings developed for other steel grades or structural types is thus not generally recommendable.

In order to overcome these problems, specific work packages dedicated to (HS steel) CHS sections were included in two on-going or targeted research projects of the authors: the CIDECT-funded project "Hollopoc" on the "Overall Interaction Concept" (OIC) design of hollow sections (carried out primarily at HES Fribourg - University of Applied Sciences of Western Switzerland), and the follow-up "Hollosstab" project specifically dedicated to HS steel (with additional research partners from IST Lisbon and Imperial College London, as well as from the industry). Both projects are part of the STSS (2012) research framework, initiated by the authors.

The "Overall Interaction Concept" is explained in a second paper in these same SSRC conference proceedings (Nseir et al. 2014) and will thus not be discussed in detail here. Briefly, as the name implies, and Fig. 10a shows, the method again has many analogies with the EN 1993-1-6 "Overall Method", as well as with DSM. The focus of the method, however, is globally and locally slender beam-columns, with combined bending and compression; isolated load cases (only N, only M) and buckling modes (only local, only global) are included in the general procedure as special cases.

The "Hollosstab" project, which specifically focuses on HS steel, is currently at the preparation/initial test phase. Preliminary results for HS steel CHS, developed by means of a numerical (GMNIA) study, are shown in the following. They give an indication of the general tendencies to be expected from a more thorough experimental validation, and allow one to formulate some first proposal formats for design equations.

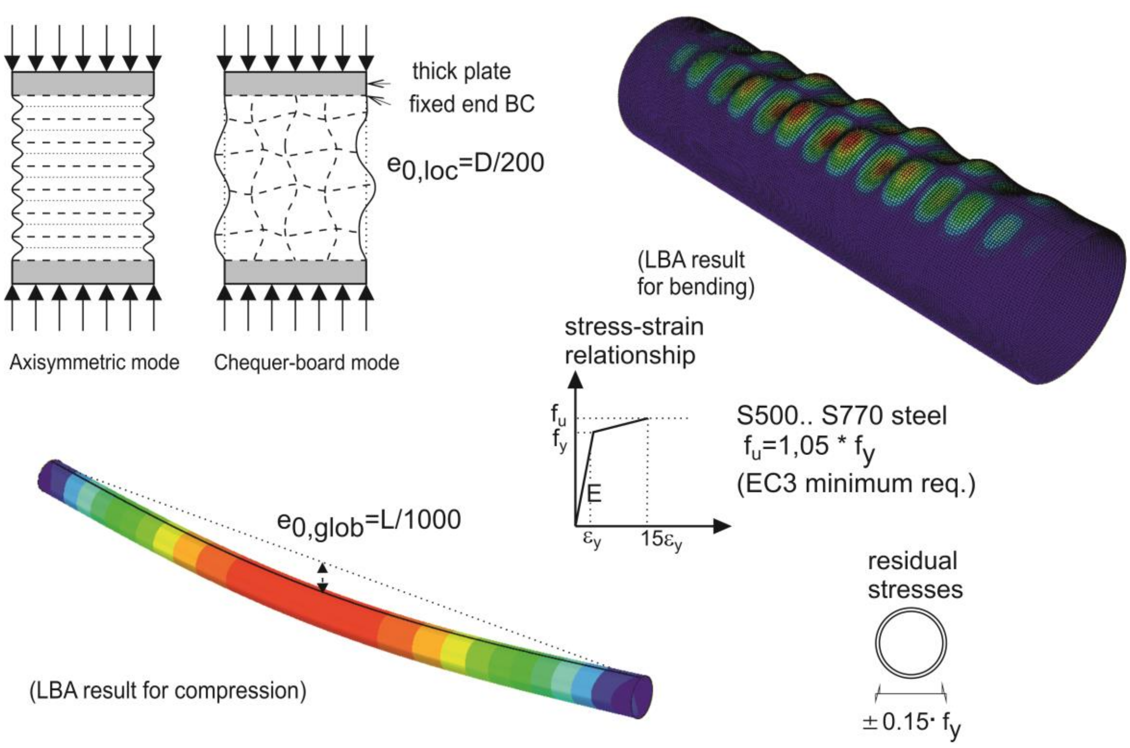


Figure 11: Methodology and assumptions for the preliminary numerical studies

4.2 Preliminary numerical studies – methodology of the GMNIA calculations.

The methodology employed in the preliminary numerical studies of this paper is summarized in Fig. 11. The following assumptions were made:

i. The shape of the lowest local buckling eigenmode, as shown by an LBA analysis, is considered to be the most detrimental local imperfection shape for the studied CHS. This is generally a sufficiently accurate assumption only for cylinders of intermediate local slenderness, as other patterns may be more critical in the case of either more thick- or thin-walled elements. The amplitude of the imperfection was chosen to have a value proportional to the diameter of the shell, in accordance with the tolerance specifications in EN 10210 and EN 10219.
ii. The global imperfection – relevant for the study of beam-column behavior – was applied with an amplitude of $e_{0,glob}$=L/1000. This corresponds to the common assumptions made during the development of the current EC3 beam-column design rules.
iii. Residual stresses were modelled with the amplitudes shown in the figure. The minimum, nominal material properties (yield stress f_y, elongation at necking ε_u=15 ε= 15 f_y/E, f_u=1.05 f_y) were used for HS steel.
iv. For both short and longer elements, a boundary condition corresponding to "thick plates" and penetration welds ("rotationally fixed shell edges at ends") was considered; for the purposes of global buckling, the beam-columns had hinged ends.
v. The applied reference load was equal to the maximum plastic capacity R_{pl} – calculated omitting any local buckling. In the case of isolated compression, this simply means that A*f_y was applied. In the case of bending, the plastic section modulus W_{pl} was used and a moment of M= W_{pl} * f_y was applied. Only a selection of results is discussed in the following due to space limitations; these results are representative of the general findings.

4.3 First results and proposals

A first series of calculation results is reported in Table 4. A number of HS steel sections with "common", small to medium diameters and steel grades up to S770 (f_y=770 MPa) were loaded in axial / meridional compression. All values of D/t exceeded the limit value of 90 ε^2 and the cross-sections were thus "slender" according to the general part 1-1 of Eurocode 3. The length of the sections was set to L=6,67 * D / ε, which leads to globally stocky members with $\bar{\lambda}_{glob}$~0.2.

Table 4: Numerical (GMNIA) results for local buckling (globally stocky CHS)-
compression, several fabricated dimensions

CHS Dimensions D x t [mm]	Steel Grade ($f_y=R_{eH,min}$)	L [mm]	$\frac{D/t}{\varepsilon^2}$	R_{cr} [-]	$\bar{\lambda}$ [-]	χ_{GMNIA} [-]
273 x 8,0	S770 – (f_y=770MPa)	1006.0	111.8	8.753	0.3380	1.004
177,8 x 5,0	S770	655.2	116.5	8.4324	0.3444	1.004
355,6 x 8,0	S770	1310.3	145.6	7.0000	0.3780	0.989
200 x 4,0	S500	914.5	106.4	9.3264	0.3274	0.9978
200 x 3,0	S500	914.5	141.8	7.1630	0.3736	0.9890
273 x 4,0	S700	1055.1	203.3	5.0607	0.4445	0.9770
244,5 x 5,0	S700	944.9	145.7	6.9812	0.3785	0.9910

The obtained values of $R_{cr,}$ $\bar{\lambda}$ and χ_{GMNIA} are shown in the table. It is evident, from these first calculations, that the studied sections had a rather small tendency to buckle in a local mode: all reduction factors are very close to 1.0.

In a second step, fictitious sections with D=300 and progressively reduced wall thickness, all of steel grade S690, were looked at in order to obtain GMNIA results with a more severe effect of local buckling. These results are summarized in Table 5

Table 5: Numerical (GMNIA) results for local buckling (globally stocky CHS)-
compression, D=300mm, variable thickness

CHS Dimensions D x t [mm]	Steel Grade ($f_y=R_{eH,min}$)	L [mm]	$\frac{D/t}{\varepsilon^2}$	R_{cr} [-]	$\bar{\lambda}$ [-]	χ_{GMNIA} [-]
300 x 10,0	S690 (f_y=690MPa)	1167.8	88.1	10.976	0.302	1.002
300 x 8,0			110.1	8.910	0.335	1.000
300 x 6,0			146.8	6.925	0.380	0.989
300 x 4,0			220.2	4.726	0.460	0.956
300 x 3,0			293.6	3.594	0.528	0.925
300 x 2,0			440.4	2.442	0.640	0.881
300 x 1,75			503.3	2.150	0.682	0.863
300 x 1,50			587.2	1.856	0.734	0.813

It was consequently attempted to describe these results with an analytical formula, based on the EN 1993-1-6 buckling reduction factors, but transferred into a format that is more suitable for the planned "OIC" type of representation.
For this purpose, as a first proposal, the well-known "Ayrton-Perry" format was chosen, as this format can also very advantageously be used for the representation of (beam-)column reduction factors. The following expressions were thus calibrated:

$$\chi_{x,loc} = \frac{1}{\Phi_{loc} + \sqrt{\Phi_{loc}^2 - \frac{\bar{\lambda}_{loc}^2}{\alpha_1}}} \leq 1.0 \qquad (11)$$

with
$$\Phi_{loc} = 0.5 \cdot \left(1 + \alpha_0 \left(\bar{\lambda}_{loc} - \bar{\lambda}_{loc,0}\right) + \frac{\bar{\lambda}_{loc}^2}{\alpha_1}\right) \qquad (12)$$

$$\alpha_0 = 0.5 \quad ; \quad \alpha_1 = 0.8 \cdot \left(1 + 2\left(w_k / t\right)^{1.5}\right) \quad ; \quad \Delta w_k / t = \frac{1}{Q}\sqrt{\frac{r}{t}} \qquad (13)$$

The plateau value $\bar{\lambda}_{loc,0}$ is at first deliberately left undefined, but will typically take values between 0.2 and 0.4. Note that $\Delta w_k/t$ is taken directly from Eq. 3b, while α_1 corresponds to a slightly modified version of Eq. 3a. The index "loc" is introduced here to distinguish the reduction factor for local buckling and its coefficients from the corresponding factors for global (column) buckling.

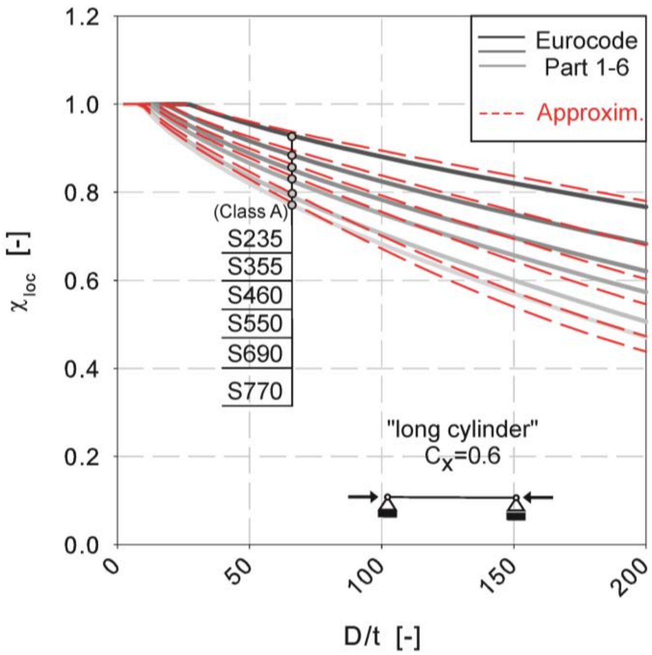


Figure 12: Approximation of the EN 1993-1-6 local buckling rules by an alternative, more compact formulation.

The results of this approximation are shown first for a purely analytical evaluation of the slenderness $\bar{\lambda}_{loc}$ =f(D/t) and of the EC3-1-6 reduction factor $\chi_x=\chi_{loc}$, using Eq. 6 and Eq. 1, C_x=0.6, and $\bar{\lambda}_{loc,0}$=0.2, see Fig. 12. The figure clearly illustrates the accuracy of the approximation.

In a subsequent step, the results of Table 5 are compared with the EC3-1-6 results and the above approximation calculated in two different ways:

i. The EC3-1-6 results were calculated via σ_{crit} obtained using the approximation for C_x given in the code, i.e. the red line in Fig. 5, with the appropriate factors for the given “fixed shell edge” boundary condition, length and D/t ratios. Furthermore, Eq. 2 and the standard coefficients in Table 1 and 2 for α, β, η and $\bar{\lambda}_0$ (all for meridional compression) were used. This leads to the dark continuous line in Fig. 13.
ii. The same procedure for the calculation of σ_{crit} was carried out for the above approximation of the EC3-1-6 local buckling rules, using Eq. 11 to 13 with $\bar{\lambda}_{loc,0}$=0.2. This is represented by the red dashed line in Fig. 13, again showing the accuracy of the approximation (“Approxim. v1”) in describing the EC3 curves – yet not quite in describing the GMNIA results.
iii. Finally, the approximation of Eq. 11 to 13 was evaluated directly on the basis of $\bar{\lambda}_{loc}$ from the numerical LBA analysis, i.e. without the need for explicitly calculating σ_{crit} from Eq. 6. Additionally, and more importantly, the plateau value of $\bar{\lambda}_{loc,0}$ was raised to $\bar{\lambda}_{loc,0}$=0.4. This leads to the “Approxim. v2” points in Fig. 13, which match the GMNIA results quite closely.

It shall be noted that a value of $\bar{\lambda}_{loc,0}$=0.4 more closely matches the D/t=90 ε^2 value of EN 1993-1-1 (and the similar limit/”plateau” values of D/t in other codes).

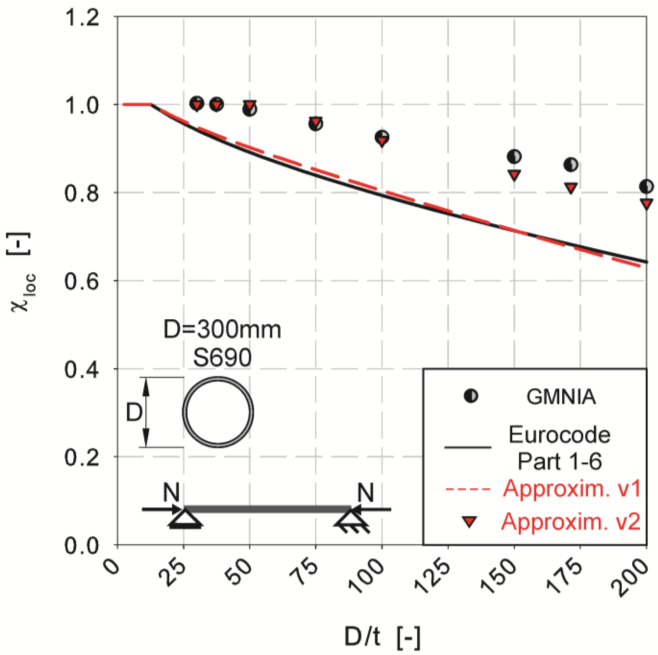


Figure 13: Local buckling of HS steel CHS in compression; GMNIA vs. analytical predictions.

Finally, some first results concerning the behavior of CHS beam-columns are presented in the following. For CHS, the main problems to be dealt with are: i. the interaction between local and global buckling, and ii. the effect of the bending moment in a representation of the beam-column strength in terms of the "Overall Interaction Concept", i.e. on the basis of a combined, overall slenderness equivalent to the one given by Eq. 1.

Both aspects are separately dealt with in the two diagrams of Fig. 14:

i. Fig. 14a treats the interaction between local and global buckling of a HS steel CHS with D/t=273/5=54.6. With steel grade S690, this section falls clearly outside of the D/t lmit value of 90 ε^2=30.7, thus making it necessary to consider local/global interactive buckling. This is done using the following equations 14 to 16, which apply the conventional EC3 approach (comparable to the "Q" factor approach of AISC).

$$\chi_{glob,gr} = \chi_{x,loc} \cdot \mathrm{MIN}\left[\frac{1}{\Phi_{glob} + \sqrt{\Phi_{glob}^2 - \bar{\lambda}_{glob}^2}};1.0\right] \tag{14}$$

with

$$\Phi_{glob} = 0.5 \cdot \left(1 + \alpha_{glob}\left(\bar{\lambda}_{glob+loc} - 0.2\right) + \bar{\lambda}_{glob+loc}^2\right) \tag{15}$$

$$\bar{\lambda}_{glob+loc} = \sqrt{\frac{\chi_{x,loc} \cdot A \cdot f_y}{\pi^2 EI / L_{cr}^2}} = \bar{\lambda}_{glob,gr} \cdot \sqrt{\chi_{x,loc}} \tag{16}$$

In the above equations, the $\chi_{glob,gr}$ is the global reduction factor, including effects of local buckling, defined as the reduction of the gross cross-sectional resistance A*f_y. $\chi_{x,loc}$ is the purely local reduction factor defined above.

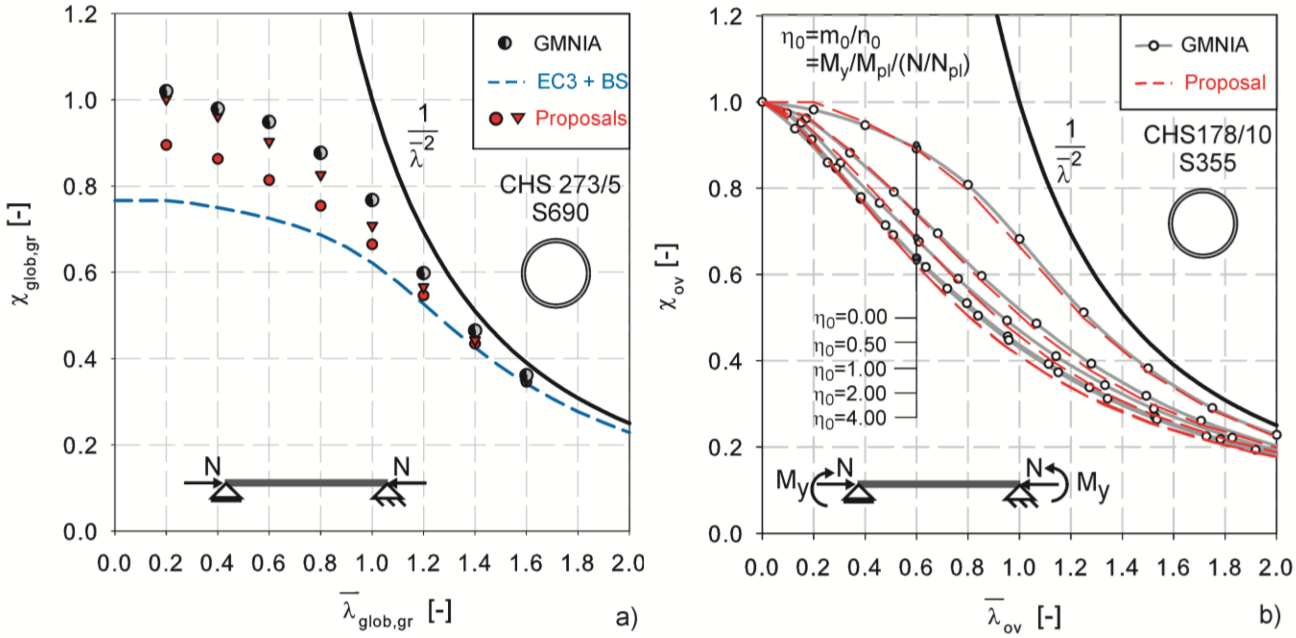


Figure 14: Buckling strength of beam-columns using the "Overall Interaction Concept"; locally and globally slender column (a); beam-column loaded by eccentric axial forces (b).

Two proposals are made in Fig. 14a order to describe the GMNIA results: both make use of the "exact" numerical slenderness retrieved from the LBA analysis, but the lower of the two proposals uses $\bar{\lambda}_{loc,0}$=0.2, while the higher one uses $\bar{\lambda}_{loc,0}$=0.4. The increased accuracy of the latter can again be appreciated. These proposals are compared to the existing EC3 rule, combined with the simple effective area formula of BS 5950-1, Eq. 7. The resulting curve is plotted as a blue dashed line in Fig. 14a.

ii. Fig. 14b deals with the behavior of relatively stocky CHS beam-columns, loaded by eccentric axial forces at both ends, resulting in compression plus a uniform first-order bending moment diagram. The GMNIA results are shown by the grey line with circle marks. A proposal was made in (Taras 2010; Taras & Greiner 2010) which leads to a very accurate description of the beam-column's buckling strength in terms of the "overall" slenderness and reduction factor $\bar{\lambda}_{ov}$ and χ_{ov}. While the details cannot be included in this paper, it shall be mentioned that the key feature of the formulation is the inclusion of the "load eccentricity", normalized according to Eq. 17 by the plastic section core width M_{pl}/N_{pl}, into the Ayrton-Perry imperfection term $\alpha_{glob}\left(\bar{\lambda}_{glob+loc}-0.2\right)$ of Eq. 15.

$$\eta_0=\frac{\left(M/M_{pl}\right)}{\left(N/N_{pl}\right)}=\frac{m_0}{n_0} \tag{17}$$

Note that no local buckling is yet included in this proposal. However, the results for stockier beam-columns and for locally slender columns are encouraging for the feasibility of a combination of the two solutions.

5. Summary, Conclusions and Outlook

In the first part of this paper, the Eurocode 3 – Part 1-6 "Overall Method" for the design of steel shell structures against buckling was described in this paper and compared to other methods that share the same basic concepts, namely the use of numerical analysis techniques for the precise calculation of the elastic bifurcation load of the studied system and the application of a global buckling reduction factor to the (plastic) collapse load of the system. The analogies and differences between this design method for thin-walled shell structures and the Direct Strength Method (DSM) for thin-walled, cold-formed "flat" cross-sections were discussed.

The second part of the paper discussed the need for the consideration of shell buckling resistances in the design of only moderately thin-walled circular hollow sections (CHS) made of high-strength steel with yield stress values of up to and beyond S770. These sections may often fall below the common slenderness limits D/t between semi-compact and slender cross-sections. It is therefore necessary to account for local buckling, as well as global buckling whenever the studied member is also susceptible to global, flexural column buckling.

Two on-going or starting research projects were presented, which deal specifically with the resistance of slender, high-strength CHS, making use of the "Overall Interaction Concept", a concept for the design of slender beam-columns that is comparable to the DSM. Initial results were shown, based on preliminary numerical studies, and possible formats for design equations were presented. The mentioned research projects will seek to answer the following open questions concerning the design of slender, high-strength CHS, namely:

i. What is the influence of the characteristic, low-ductility stress-strain curve of HS steels on the applicable slenderness limits for the on-set of elasto-plastic buckling (semi-compact to slender cross-section transition) of CHS?

ii. In the semi-compact and compact range: is enough ductility present to allow for (partial) plastic design of HS steel CHS members?

iii. Can the concept of classification of cross-sections – and the corresponding non-continuous strength curves for members of increasing local slenderness – be replaced by a continuous strength curve, as function of the "Overall Slenderness", i.e. using the "Overall Interaction Concept"?

iv. Can the design of locally and globally slender HS steel hollow-section beam-columns be accommodated within the OIC? Can variable bending moment diagrams, respectively stress gradients in longitudinal direction, be incorporated into easy-to-use design rules?

v. Are the local imperfection amplitudes which are currently explicitly or implicitly contained in existing buckling design rules for cylindrical structures representative of the scatter-band of actual geometric shape deviations in CHS fabricated to EN 10210, EN 10219 or comparable other international standards, or can reduced imperfections be used in design?

vi. Do the lower strength reserves on the yield stress (actual vs. nominal, minimum value) to which HS steel grades are produced have a detrimental impact on the safety factors to be used in design?

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